Entanglement swapping: entangling atoms that never interacted

Carlos Renato de Carvalho, and Emerson S. Guerra[*]

Instituto de Física - Universidade Federal do Rio de Janeiro
Cx.Postal 68528, 21945-970 Rio de Janeiro, RJ, Brazil
[*]Departamento de Física - Universidade Federal Rural do Rio de Janeiro
Cx. Postal 23851, 23890-000 Seropédica, RJ, Brazil

Abstract

In this work we discuss a proposal of entangling atomic states of particles which have never interacted. The experimental realization proposed makes use of the interaction of Rydberg atoms with a micromaser cavity prepared in a coherent state.

Usually entanglement is understood as a consequence of some interaction of the particles in their common past. Thus far, it has been achieved either by having the two particles emerging from the same source [1] or by having the two particles interacting with each other [2]. However, Yurke and Stoler [3] and Zukowski *et al* [4] showed that one can entangle particles that do not even share any common past. In a recent paper it has been suggested an experimental realization of teleportation of atomic states via cavity quantum eletrodynamics [5]. Based on the results presented there, in the present work we show that it is possible to build a scheme to entangle the states of two atoms which have never interacted.

Consider a three-level cascade atom A_k with $|h_k\rangle, |e_k\rangle$ and $|g_k\rangle$ being the higher, intermediate and lower atomic states. Our scheme involves Ramsey cavities and another one (C) with high quality factor. We assume that the transition $|e_k\rangle \rightleftharpoons |f_k\rangle$ is far enough from resonance with the field inside C, so that the dispersive interaction between atom and field can be described by the time evolution operator

$$U = e^{-i\varphi(a^\dagger a+1)}|h_k\rangle\langle h_k| + e^{i\varphi a^\dagger a}|e_k\rangle\langle e_k|, \tag{1}$$

where a ($a^\dagger$) is the annihilation (creation) operator for the field in C, $\varphi = g^2\tau/\ \Delta$, g is the coupling constant, τ is the interaction time, $\Delta = \omega_h - \omega_e - \omega$ is the detuning, ω_h and ω_e are the frequencies of the upper and intermediate levels respectively and ω is the cavity C frequency. In addition we assume that the transitions $|e_k\rangle \rightleftharpoons |g_k\rangle$ and $|h_k\rangle \rightleftharpoons |g_k\rangle$ are highly detuned from the ω so that there will be no coupling with the cavity field involving the state $|g_k\rangle$. However we suppose that $|g_k\rangle$ is coupled to the state $|e_k\rangle$ in the Ramsey cavities which we shall use to prepare the atomic Bell states involving the states $|e_k\rangle$ and $|g_k\rangle$. Therefore, considering the atom-field interaction in C, the level $|h_k\rangle$ will never be populated during the whole process so that we can ignore it from now on, since it will not play any role in our scheme, being important only as origin of the phase factor in the time evolution operator (see Eq.(2)). Hence, we have effectively a two-level system involving the states $|e_k\rangle$ and $|g_k\rangle$, corresponding to the time evolution operator

$$U = e^{i\pi a^\dagger a}|f_k\rangle\langle f_k| + |g_k\rangle\langle g_k|, \tag{2}$$

where the second term above was put by hand just in order to take into account the effect of level $|g_k\rangle$ and we have taken $\varphi = \pi$.

Now, we discuss briefly how to prepare two entangled atoms. First, we assume that we have an atom A_1, initially in the state $|\psi\rangle_{A_1} = \frac{1}{\sqrt{2}}(|f_1\rangle + |g_1\rangle)$, which passes through C (prepared in coherent state $|-\alpha\rangle$), whose time evolution of the system formed by the atom and the cavity field is ruled by Eq.(2). After that, we let atom A_1 pass through a Ramsey cavity R_0, where the atomic states are rotated according to

$$\frac{1}{\sqrt{2}}(|f_1\rangle + |g_1\rangle) \rightarrow |f_1\rangle, \qquad \frac{1}{\sqrt{2}}(-|f_1\rangle + |g_1\rangle) \rightarrow |g_1\rangle. \tag{3}$$

Second, we let another atom A_2 cross the cavity C and also the Ramsey cavity R_0, so that it is subjected to the same effects represented by Eqs. (2) and (3). In the case of the Ramsey cavity, we supose that it, as a classical device, can produce the rotation (3) in A_1 as well as in A_2. Now, we inject $|-\alpha\rangle$ in cavity

C, which mathematically is represented by the operation $D(\beta)|\alpha\rangle = |\alpha+\beta\rangle$, where $D(\beta) = e^{(\beta a^\dagger - \beta^* a)}$, and experimentally it is obtained with a classical oscillating current in an antenna coupled to the cavity. At this point, the system state is given by

$$|\psi\rangle_{A_1-A_2-C} = \frac{1}{2}\Bigg\{|f_1\rangle|f_2\rangle\Big(\ |0\rangle + |-2\alpha\rangle\ \Big) - |g_1\rangle|g_2\rangle\Big(\ |0\rangle - |-2\alpha\rangle\ \Big)\Bigg\}. \qquad (4)$$

In order to disentangle the atomic states of the cavity field state, we now send a two-level atom A_3, resonant with the cavity, with $|f_3\rangle$ and $|e_3\rangle$ being the lower and upper levels respectively, through C. A_3 is sent in the lower state $|f_3\rangle$. When interacting with the cavity field, we know that the state $|f_3\rangle|0\rangle$ does not evolve, whereas the state $|f_3\rangle|-2\alpha\rangle$ evolves to $|e_3\rangle|\chi_e\rangle + |f_3\rangle|\chi_f\rangle$, where $|\chi_e\rangle$ and $|\chi_f\rangle$ are different cavity field states, whose expressions, in terms of the Fock states, do not concern us, but which are well known under the Jaynes-Cummings model [6]. Finally, the last step to obtain the entangled state of atoms A_1 and A_2 is detecting A_3 in the state $|e_3\rangle$. It yields

$$|\Phi^+\rangle_{A_1-A_2} = \frac{1}{\sqrt{2}}(|f_1\rangle|f_2\rangle + |g_1\rangle|g_2\rangle). \qquad (5)$$

Following a similar proceeding, we can form a Bell basis [7]

$$\begin{aligned} |\Phi^-\rangle_{A_1-A_2} &= \frac{1}{\sqrt{2}}(|f_1\rangle|f_2\rangle - |g_1\rangle|g_2\rangle), \\ |\Psi^-\rangle_{A_1-A_2} &= \frac{1}{\sqrt{2}}(|f_1\rangle|g_2\rangle - |g_1\rangle|f_2\rangle), \\ |\Psi^+\rangle_{A_1-A_2} &= \frac{1}{\sqrt{2}}(|f_1\rangle|g_2\rangle + |g_1\rangle|f_2\rangle), \end{aligned} \qquad (6)$$

which is a complete orthonormal basis for atoms A_1 and A_2.

Consider that we have a system, which consists of two pairs of entangled atoms, whose state is

$$|\Psi\rangle_{A_1-A_2-A_3-A_4} = |\Psi^-\rangle_{A_1-A_2} \otimes |\Psi^-\rangle_{A_3-A_4}. \qquad (7)$$

The above state can be rewritten as

$$\begin{aligned} |\Psi\rangle_{A_1-A_2-A_3-A_4} = \frac{1}{2}\Big(&|\Psi^+\rangle_{A_1-A_4}|\Psi^+\rangle_{A_2-A_3} + |\Psi^-\rangle_{A_1-A_4}|\Psi^-\rangle_{A_2-A_3} + \\ &|\Phi^+\rangle_{A_1-A_4}|\Phi^+\rangle_{A_2-A_3} + |\Phi^-\rangle_{A_1-A_4}|\Phi^-\rangle_{A_2-A_3}\Big), \end{aligned} \qquad (8)$$

Although atoms A_1 and A_4 have never interacted, we can entangle them if we measure properly the state of atoms A_2 and A_3. This is what Eq. (8) tell us. Now, we discuss how to perform the measurements in order to project the state of A_1 and A_4 onto any of the Bell states.

Let us assume we have a cavity prepared in a coherent state $|\alpha\rangle$. Notice that, if we send atoms A_2 and A_3 through C in one of the Bell states, Eqs.(5) and (6), according to the time evolution operator (2), we get

$$|\Phi^\pm\rangle_{A_2-A_3}|\alpha\rangle \longrightarrow |\Phi^\pm\rangle_{A_2-A_3}|\alpha\rangle \quad \text{and} \quad |\Psi^\pm\rangle_{A_2-A_3}|\alpha\rangle \longrightarrow |\Psi^\pm\rangle_{A_2-A_3}|-\alpha\rangle \qquad (9)$$

Therefore, considering (8), after atoms A_2 and A_3 pass through the cavity and, then, we inject $|\alpha\rangle$ in C, it yields

$$\begin{aligned} |\Psi\rangle_{A_1-A_2-A_3-A_4-C} = \frac{1}{2}\Big(&|\Psi^+\rangle_{A_1-A_4}|\Psi^+\rangle_{A_2-A_3}|0\rangle + |\Psi^-\rangle_{A_1-A_4}|\Psi^-\rangle_{A_2-A_3}|0\rangle + \\ &|\Phi^+\rangle_{A_1-A_4}|\Phi^+\rangle_{A_2-A_3}|2\alpha\rangle + |\Phi^-\rangle_{A_1-A_4}|\Phi^-\rangle_{A_2-A_3}|2\alpha\rangle\Big), \end{aligned} \qquad (10)$$

As done before, in order to disentangle the atomic states of the cavity field state, we send a two-level atom A_5, resonant with the cavity, with $|f_5\rangle$ and $|e_5\rangle$ being the lower and upper levels respectively, through C. If A_5 is sent in the lower state $|f_5\rangle$, after we detect A_5 in $|e_5\rangle$, we get

$$|\Psi\rangle_{A_1-A_2-A_3-A_4} = \frac{1}{\sqrt{2}}\Big(|\Phi^+\rangle_{A_1-A_4}|\Phi^+\rangle_{A_2-A_3} + |\Phi^-\rangle_{A_1-A_4}|\Phi^-\rangle_{A_2-A_3}\Big). \qquad (11)$$

Now, we have to distinguish $|\Phi^+\rangle_{A_2-A_3}$ from $|\Phi^-\rangle_{A_2-A_3}$. In order to do this we notice that, defining the operator $\Sigma_x = \sigma_x^2\sigma_x^3$, where $\sigma_x^k = |f_k\rangle\langle g_k| + |g_k\rangle\langle f_k|$, we have $\Sigma_x|\Phi^\pm\rangle_{A_2-A_3} = \pm|\Phi^\pm\rangle_{A_2-A_3}$. Therefore, we can distinguish between $|\Phi^+\rangle_{A_2-A_3}$ and $|\Phi^-\rangle_{A_2-A_3}$ performing measurements of Σ_x. In order to do so, we proceed as follows. We make use of another Ramsey cavity, represented by the operator $R_1 = \frac{1}{\sqrt{2}}(|f\rangle\langle f| - |f\rangle\langle g| + |g\rangle\langle f| + |g\rangle\langle g|)$, to gradually unravel the Bell states. The eigenvectors of the operators σ_x^k are $|A_k^\pm\rangle = \frac{1}{\sqrt{2}}(|f_k\rangle \pm |g_k\rangle)$ and we can rewrite the Bell states as

$$|\Phi^\pm\rangle_{A_2-A_3} = \frac{1}{2}\left[|A_2^+\rangle\Big(|f_3\rangle \pm |g_3\rangle\Big) + |A_2^-\rangle\Big(|f_3\rangle \mp |g_3\rangle\Big)\right], \tag{12}$$

Let us take, for instance, $|\Phi^+\rangle_{A_2-A_3} = \frac{1}{\sqrt{2}}(|f_2\rangle|f_3\rangle + |g_2\rangle|g_3\rangle)$. Applying R_1 to this state, we have

$$R_1|\Phi^+\rangle_{A_2-A_3} = \frac{1}{2}\left\{|f_2\rangle\Big(|f_3\rangle - |g_3\rangle\Big) + |g_2\rangle\Big(|f_3\rangle + |g_3\rangle\Big)\right\}. \tag{13}$$

Now, we compare (13) and (12). We see that the rotation by R_1 followed by the detection of $|g_2\rangle$ corresponds to the detection of the state $|A_2,+\rangle$, whose eigenvalue of σ_x^2 is $+1$. After we detect $|g_2\rangle$, we get $|\psi\rangle_{A_3} = \frac{1}{\sqrt{2}}(|f_3\rangle + |g_3\rangle)$ that is, we obtain $|\psi\rangle_{A_3} = |A_3,+\rangle$. If now we apply R_1 to A_3, that is, if A_3 pass in a Ramsey cavity which performs the operation described by R_1, we get $R_1|\psi\rangle_{A_3} = |g_3\rangle$. We see that the rotation by R_1 followed by the detection of $|g_3\rangle$ corresponds to the detection of the state $|A_3,+\rangle$ whose eigenvalue of σ_x^3 is $+1$. Consequently, after this proceeding, the atoms A_1 and A_4 are collapsed in the entangled state $|\Phi^+\rangle_{A_1-A_4}$. Similarly, we can measure the eigenvalue -1 of Σ_x and, consequently, make the atoms A_1 and A_4 to collapse in the entangled state $|\Phi^-\rangle_{A_1-A_4}$.

In fact, we can change the proceeding described above, in order to make the atoms A_1 and A_4 to collapse in any state of the Bell basis (5) and (6). Summarizing, we have the following possible proceedings which results in one of the four Bell states involving atoms A_1 and A_4, which are presented in the table below:

$$\begin{aligned}
&(\text{injection of } |\alpha\rangle)(R_1,\langle g_2|)(R_1,\langle g_3|) \longleftrightarrow |A_2,+\rangle|A_3,+\rangle \Longrightarrow |\Phi^+\rangle_{A_1-A_4}\\
&(\text{injection of } |\alpha\rangle)(R_1,\langle g_2|)(R_1,\langle f_3|) \longleftrightarrow |A_2,+\rangle|A_3,-\rangle \Longrightarrow |\Phi^-\rangle_{A_1-A_4}\\
&(\text{injection of } |-\alpha\rangle)(R_1,\langle f_2|)(R_1,\langle f_3|) \longleftrightarrow |A_2,-\rangle|A_3,-\rangle \Longrightarrow |\Psi^+\rangle_{A_1-A_4}\\
&(\text{injection of } |-\alpha\rangle)(R_1,\langle g_2|)(R_1,\langle f_3|) \longleftrightarrow |A2,+\rangle|A_3,-\rangle \Longrightarrow |\Psi^-\rangle_{A_1-A_4}
\end{aligned} \tag{14}$$

Finally, let us analyze the feasibility of the experimental implementation of the above scheme of entanglement swapping. Considering Rydberg atoms of principal quantum numbers 50 or 51, the radiative time is of the order of 10^{-2} s and the coupling constant g is of the order of $2\pi \times 25$ kHz [8, 9, 10] and the detuning Δ is of the order of $2\pi \times 100$ kHz. Taking into account that $\varphi = g^2\tau/\Delta$, for $\varphi = \pi$ we have an interaction time $\tau = 8 \times 10^{-5}$ s and we could, in principle, assume a time of the order of 10^{-4} s to realize the entanglement swapping which is much shorter than the radiative time. We have to consider also the cavity decay time which in recent experiments, with niobium superconducting cavities at very low temperature and quality factors in the $10^9 - 10^{10}$ range, have a cavity energy damping time of the order of 10 to 100 ms, and which could be larger than the required time to perform the entanglement swapping.

References

[1] S. J. Freedman and J. S. Clauser, Phys. Rev. Lett. **28**, 938 (1972); A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. **49**, 91 (1982).

[2] M. Lamehi-rachti and W. Mittig, Phys. rev D **14**, 2543 (1976); E. Hagley, X. Maitre, G. Nogues, C. Wunderlich, M. Brune, J.M. Raimond, and S. Haroche, Phys. Rev Lett **79**, 1 (1997).

[3] B. Yurke and D. Stoler, Phys. Rev. A **46**, 2229 (1992); Phys.Rev. Lett. **68**, 1251 (1992).

[4] M. Zukowski, A. Zeilinger, M. A. Horne, and A. K. Ekert, Phys. Rev. Lett. **71**, 4287 (1993).

[5] E. S. Guerra, Opt. Commun. **242**, 541 (2004).

[6] M. Orszag, *Quantum Optics*, (Springer-Verlag, Berlin, 2000).

[7] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, (Cambridge Univ. Press, Cambridge, 2000); S. L. Braustein, A. Mann and M. Revzen, Phys. Rev. Lett. **68**, 3259 (1992).

[8] A. Rauschenbeutel, G. Nogues, S. Osnaghi, P. Bertet, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **83**, 5166 (1999).

[9] A. Rauschenbeutel, G. Nogues, S. Osnaghi, P. Bertet, M. Brune, J. M. Raimond, and S. Haroche, Science 2**88**, 2024 (2000).

[10] P. Bertet, A. Auffeves, P. Maioli, S. Osnaghi, T. Meunier, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **89**, 200402 (2002).