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Phases of the Waves Diffracted by Relief Gratings and the Energy Conservation in Wave Mixing Experiments

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Abstract

In this paper we analyze the wave mixing between the waves diffracted by a relief grating, in order to understand the relation between the phases of the diffracted waves and the geometrical parameters of the grating. Theoretical calculations of these phases as well as experimental measurements were presented demonstrating the validity of the analysis.

Introduction

Diffraction gratings are well known devices used to separate the spectrum of the light. In a medium of refractive index n_1, the directions of the diffracted waves (β_m) are given by the famous grating equation:

$$d \cdot (\sin\alpha + n_1 \cdot \sin\beta_m) = m \cdot \lambda \tag{1}$$

being: **d** the grating period, λ the incident wavelength, **m** the number of the diffracted order, α the incident angle and $\boldsymbol{\beta_m}$ the angle of the diffracted **m** order. The signal of the angles and of the diffracted orders are defined in the Figure 1

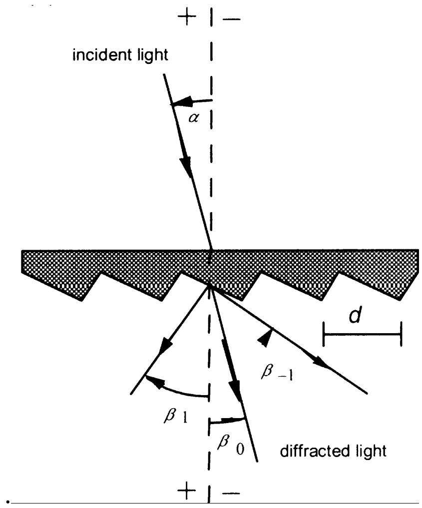

Figure 1: Scheme of the directions of the diffracted orders

The amplitude of the diffracted waves, however, depend not only of the incident wavelength and the grating period, but also on the polarization of the incident wave and on the depth and shape of the grooves. If the grating period is of the same magnitude of the incident wavelength (resonant domain), it is necessary to use rigorous diffraction theories to calculate the diffracted waves. In this region the gratings present interesting polarization properties that can be used to fabricate diffractive polarizing elements.

The diffraction calculation using rigorous diffraction theories is fully numerical rising difficult to understand the general behavior of the diffracted waves. Although such phases are very important to design polarizing elements as wave-plates, the role of the phases of the diffracted waves is few addressed in the literature. Otherwise, if there is no interference between the diffracted waves, these phases have no physical meaning.

In this work we analyze an experiment of wave mixing between the waves diffracted by a relief grating, in order to understand the relation between the phases of the diffracted waves and the geometrical parameters of the grating.

The wave mixing of the diffracted waves

We assume a relief grating (of period d) illuminated simultaneously by two symmetrical waves (E_A and E_B) of the same wavelength at the Littrow (or Bragg) incident angle. Figure 2 illustrates such situation. At this condition the sinus of the incident angle in Eq. (1) is given by:

$$\sin\alpha = \pm\frac{\lambda}{2d} \tag{2}$$

The minus signal corresponds to the upper incident wave and the plus signal the bottom incident wave (in Figure 2). If we call **k** the number of the diffracted order of the upper wave and **m** the number of the diffracted order of the bottom wave, we can observe that the diffracted directions β_m and β_k will coincide if :

$$m - k = 1 \tag{3}$$

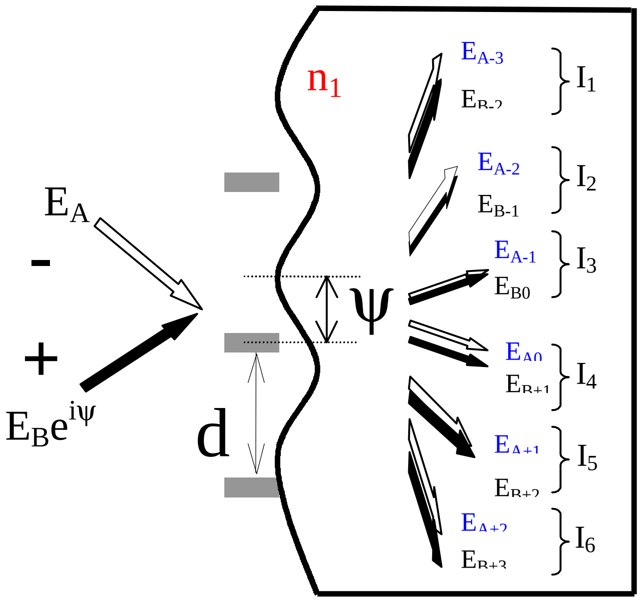

Figure 2: Scheme of the wave mixing between the waves diffracted in a relief grating.

The transmitted light is distributed in the directions given by Eq. (1). In each of these directions there is wave mixing between two consecutive orders of the symmetrically incident beams. For the directions shown in the Figure 2 the intensities resulting of the interference of each couple of waves is given by:

$$\begin{aligned}
I_1 &= I_{B-2} + I_{A-3} + 2\sqrt{I_{B-2}I_{A-3}}\,cos\left(\psi + \varphi_{B-2} - \varphi_{A-3}\right)\\
I_2 &= I_{B-1} + I_{A-2} + 2\sqrt{I_{B-1}I_{A-2}}\,cos\left(\psi + \varphi_{B-1} - \varphi_{A-2}\right)\\
I_3 &= I_{B0} + I_{A-1} + 2\sqrt{I_{B0}I_{A-1}}\,cos\left(\psi + \varphi_{B0} - \varphi_{A-1}\right)\\
I_4 &= I_{A0} + I_{B+1} + 2\sqrt{I_{A0}I_{B+1}}\,cos\left(\psi + \varphi_{B+1} - \varphi_{A0}\right)\\
I_5 &= I_{A+1} + I_{B+2} + 2\sqrt{I_{A+1}I_{B+2}}\,cos\left(\psi + \varphi_{B+2} - \varphi_{A+1}\right)\\
I_6 &= I_{A+2} + I_{B+3} + 2\sqrt{I_{A+2}I_{B+3}}\,cos\left(\psi + \varphi_{B+3} - \varphi_{A+2}\right)
\end{aligned} \tag{4}$$

With ψ being the phase difference between the incident waves (E_A and E_B in Fig.2) and the **φs** being the phases added by the diffraction, whose values depend on the diffraction order number. $\mathbf{E_{Ak}}$ and $\mathbf{E_{Bm}}$ are the amplitude of the diffracted orders and $\mathbf{I_{Ak}}$ and $\mathbf{I_{Bm}}$ their corresponding intensities. The total number of existing orders depend on the grating period (given by Eq. 1). For larger periods more orders are present.

Due to the symmetry of the incident beams:

$$\varphi_{A+n} = \varphi_{B-n} \tag{5}$$

and also for the amplitude of the diffracted waves:

$$E_{A+n} = E_{B-n} \tag{6}$$

As it can be observed from Eqs. (4), the intensities in each diffraction direction depends on the phase differences of the diffracted orders $\varphi_{Bn} - \varphi_{An-1}$ and of the phase $\boldsymbol{\psi}$. This last phase representing the actual phase shift between the incident waves which also represents the lateral shift between the grating and the interference pattern formed by the incident waves. However, the sum of the total transmitted intensities can not depend on this phase ψ because this sum represents the total transmitted energy, and it may not depend on the lateral position of the grating.

If the period of the grating in Eq. 1 satisfies:

$$(\lambda / 2 + n_1 \cdot d) \langle 2\lambda \tag{7}$$

there is only one diffracted order for each incident beam, thus Eq. (4) reduces to only two directions whose intensities are:

$$\begin{aligned} I_A &= I_{A0} + I_{B+1} + 2\sqrt{I_{A0} I_{B+1}}\, cos(\psi + \varphi_{B+1} - \varphi_{A0}) \\ I_B &= I_{B0} + I_{A-1} + 2\sqrt{I_{B0} I_{A-1}}\, cos(\psi + \varphi_{B0} - \varphi_{A-1}) \end{aligned} \tag{8}$$

Thus, in order to keep the sum of the intensities independent of the phase $\boldsymbol{\psi}$, the phase difference:

$$\varphi_{B+1} - \varphi_{A0} = \varphi_{A-1} - \varphi_{B0} \equiv \pi / 2 \tag{9}$$

Thus the phase difference between the first and the zeroth diffracted orders must be equal to $\pi/2$, independently of the grating shape or depth..

When the period of the grating allows the presence of higher diffraction orders, the phase difference $\boldsymbol{\varphi_1\text{-}\varphi_0}$ is no longer restrict to $\pi/2$, but it can vary because there are more diffracted directions to be summed. These phase variations must satisfy the condition that the sum of the intensities in all diffracted directions must be independent of $\boldsymbol{\psi}$. In order to check these conclusions theoretical calculations and experimental measurements of the phase difference $\boldsymbol{\varphi_1\text{-}\varphi_0}$ were performed.

Results and Discussions

Theoretical calculations of the phase difference between the first and zeroth diffracted orders ($\boldsymbol{\varphi_1\text{-}\varphi_0}$) were performed, using the coordinate transformation method (the C method)[1], for incidence at first-order Littrow mounting, at the wavelength 0.4579 μm, assuming a sinusoidal profiled relief grating . Figure 3 shows the phase difference $\boldsymbol{\varphi_1\text{-}\varphi_0}$ for two grating periods (**d** = 0.42μm and **d** = 0.8μm) as a function of the grating depth **h**. At small depths this curve should starts from $\pi/2$, independently of the grating profile, as expected for shallow phase gratings from the scalar theory[2]. As it can be seen from the curves, $\boldsymbol{\varphi_1\text{-}\varphi_0}$ is constant and equal to $\pi/2$ for all grating depths for the period of **d** = 0.42μm while for the period of **d** = 0.8μm it decreases with the grating depth.

In the same Figure 3 are shown the experimental measurements of such phases for some samples with **d** = 0.42 and 0.8μm with different depths. The phase measurements were performed using a method recently developed in our laboratory[3].

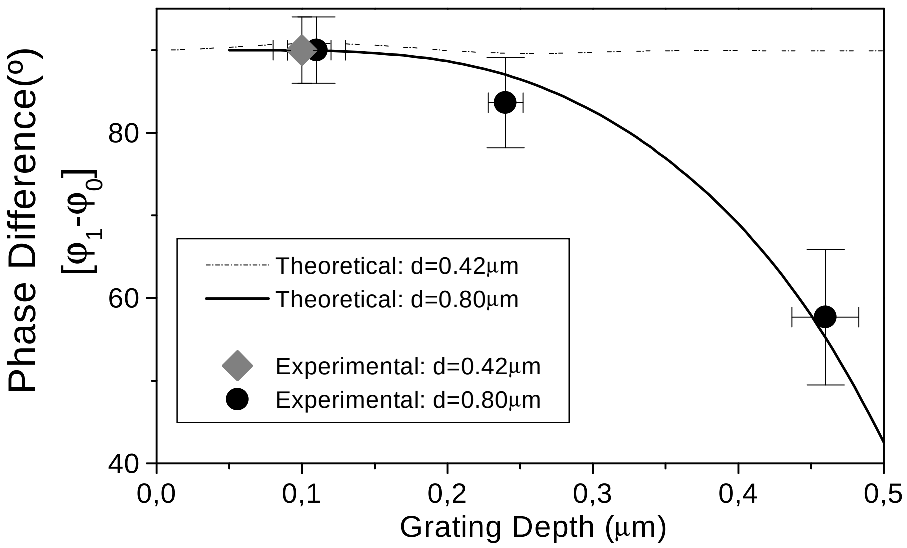

Figure 3. Theoretical and experimental values of the phase difference φ_1-φ_0 as a function of the grating depth for two periods d = 0.42μm and d = 0.8μm. The theoretical calculations assumed a sinusoidal profiled grating

Conclusions

The results presented in Figure 3 demonstrates that for a grating period **d** = 0.42μm, which satisfies Eq. (7), the phase difference $\boldsymbol{\varphi_1}$-$\boldsymbol{\varphi_0}$ = $\pi/2$ independently of the grating depth. For the grating period **d** = 0.80μm, the phase difference $\boldsymbol{\varphi_1}$-$\boldsymbol{\varphi_0}$ decreases with the grating depth. Although for the period **d** = 0.42μm there is only one experimental measurement the good agreement between the experimental measurements and the calculated phase difference curve for **d** = 0.80μm gives confidence to the theoretical calculations.

Acknowledgements

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