Moiré-like patterns due to photorefractive sinusoidal phase gratings superposition

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Abstract

Fourier synthesis of dynamic Moiré-like patterns generation with square intensity profile obtained by the superposition of more than two dynamic sinusoidal phase gratings is described. The sinusoidal phase gratings of slightly different pitches are involved and occur in the photorefractive crystal $Bi_{12}TiO_{20}$ (BTO). This holographic dynamic medium has been used in anisotropic two wave mixing architecture in diffusion only recording mechanism.

Introduction

Moiré-like patterns can be obtained from diffracted light interaction between whatever two regular periodic superimposed structures. The superposition of Ronchi rulers with slightly different superimposed pitches, for example, produces regularly spaced light fringes that have been observed and used in many optical applications [1]. Moiré patterns have been traditionally employed in optical metrology, as non-destructive testing in mechanical engineering[2,3], for example. For this, they have received a great number of new non-destructive testing analysis and been considered one of the most active technological areas.

Dynamic sinusoidal phase gratings are produced in materials exhibiting photorefractive effect [4], as a well-controlled combination of photoconductivity and electrooptic effect [5]. As a result, the real time Moiré-like patterns have been obtained by the superposition of two rotated sinusoidal dynamic phase gratings with high spatial frequency [6] rather than with permanent amplitude low spatial frequency Ronchi rulers.

In the present paper, it is proposed that the sinusoidal phase gratings of slightly different pitches, with high spatial frequencies (~1,000 lines mm^{-1}), are experimentally superimposed to produce the dynamic Moiré-like patterns. The dynamic holographic procedure exploits the nonlinear photorefraction process in the photorefractive crystal $Bi_{12}TiO_{20}$ (BTO). This holographic dynamic medium has been used in anisotropic two wave mixing architecture in diffusion only recording mechanism. This kind of pattern generation is quite interesting and exhibits a great potentiality to turn itself useful in metrological applications. The physical ideas, experiment and results will be presented and discussed in the next sections.

Experimental Setup and Procedure

Any periodic structure can be simply represented by a function like [7]

$$f(x,y) = c\,n \quad , \qquad (1)$$

where c is a constant and n is an integer that defines an order in the related fringe pattern.

Moiré-like patterns can be obtained from the diffracted light interaction between any two regular periodic superimposed structures. The superposition of two or more periodic structures given by Eq(1) is

$$\frac{f_1(x,y)}{c_1} \pm \frac{f_2(x,y)}{c_2} \ldots \pm \frac{f_j(x,y)}{c_j} \ldots = m \quad , \qquad (2)$$

where $\mathbf{c_1}$, $\mathbf{c_2}$ and $\mathbf{c_j}$ are constants and **m** is the integer that defines each equivalent generated fringe pattern order. This last expression represents the Moiré pattern structure superposition. Two Ronchi rulers,for example, is simply represented by $x = nc$, and having slightly different pitches, when superimposed, produce regularly spaced light fringes that are easily observed [7].

According to photorefractive effect theory [4], a dynamic holographic sinusoidal phase grating is obtained after illuminating the BTO sample by the interference pattern given by

$$I(x) = I_o\,(1 + m \cos k_g x) . \qquad (3)$$

When the grating reaches the highest diffraction efficiency, by cutting one of the writing beams with a mechanical shutter, the grating decay will occur and the sample will be ready to be illuminated by a new interference pattern, that is, a new grating will be produced. In Eq.(3) $k_g = 2\pi / \Lambda$ (where Λ is the grating spacing) is the grating wave number, i.e., the grating wave vector module, and m (m ~ 1, in the experiment) is the fringe modulation rate in the interference pattern.

The photorefractive BTO ($Bi_{12}TiO_{20}$) used in the present work has high electro-optic coefficient to use in CW He-Ne laser, and lower optical activity than BSO($Bi_{12}SiO_{20}$) and BGO($Bi_{12}GeO_{20}$), which allows the use of up to 1cm of thickness samples (8 mm in the present case), without external applied voltage (diffusion-only operation). That means diffraction efficiency around 5% to incident angles around 23^o (highly selected diffraction Bragg condition) to generate high spatial frequency gratings up to 1000 lines mm^{-1} in two wave mixing without distortion. Higher harmonic generation never was experimentally observed by us using beam modulation rates not restricted by m<<1, that is, m = 0.8 and eventually higher. Therefore, this experimental behavior has allowed us, without generality loss, use m<<1 restriction in the next theoretical approach. For this reason the deduced equations satisfactorily describe the experimental results obtained in the present work.

One of the most interesting features in the photorefractive effect is: after a hologram or grating decay, the sample is ready to produce a new hologram or sinusoidal phase grating, generated in the material volume without being removed from the system. In the photorefraction [5], the photogenerated carrier displacements, with a subsequent trapping, produce a volume space charge electric field distribution E_{sc} (x). Therefore, due to the linear electrooptic effect exhibited by the photorefractive materials, there is a spatial refractive index modulation that means a photoinduced birefringence in the sample volume. This spatial refractive index modulation is given in matrix form [8] by

$$\Delta\hat{n}(x) = \frac{1}{2}[r_{41}n_0^3 E_{sc}(x)\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{bmatrix} \qquad (4)$$

where n_0 is the sample refractive index, r_{41} the electrooptic coefficient, and E_{sc} the light induced space charge electric field. Eq(4) means that a holographic sinusoidal phase grating, with a refractive index modulation

$$\Delta n = \frac{1}{2} r_{41} n_o^3 E_{sc}(x) \quad , \qquad (5)$$

has been generated in the sample volume.

The result expressed in Eq.(4), taking into account the BTO crystal optical activity and the diffracted beam polarization modification, allows anisotropic diffraction. In this case, an analyzer can easily separate writing beams from the holographically reconstructed one. By doing so, the diffraction efficiency is optimized and given by [8]

$$\eta = \left(\frac{\pi \Delta n(x)}{\lambda \cos\theta_B} m \frac{\sin(\rho l)}{\rho}\right)^2 \quad , \qquad (6)$$

where θ_B is the incident Bragg angle and λ the laser light source wavelength, ρ is the optical activity parameter and l is the crystal sample thickness. The remainder terms have already been defined.

In the present work, the anisotropic diffraction was used to obtain the reconstructed dynamic Moiré-like patterns experimentally. Many sinusoidal phase gratings of slightly different pitches were generated and superinposed one by one. This means that the the light induced space charge electric field is

$$E_{sc} = E_0[e^{ik_1 x} + e^{ik_2 x} + ... + e^{ik_N x}] \quad , \qquad (7)$$

where N is the total nunber of gratings. The real part of this field is

$$E_{sc}(x) = E_0 \cos(k_1 x)\sum_{j=0}^{N}\cos(j\Delta k)x \quad , \qquad (8)$$

In this last expression the Δk is the spatial frequency variation between the superinposed gratings in the process. Experimentally this is made by changing the incident angle of the writing beans (see **Figure.1**) for any new grating by the same amount from the first one with spatial frequency k_1. Well, according to equation (5) the index modulation now is

$$\Delta n = \Delta n_0 \cos(k_1 x)\sum_{j=0}^{N}\cos(j\Delta k)x \quad . \qquad (9)$$

This last value of the index modulation in the diffraction efficiency expression (6) explains the obtained Moiré-like patterns in our experiments (see **Figure.2**).

The Moiré-like patterns was developed using a BTO crystal sample (8 x 8 x 8 mm^3) as holographic medium. This sample is a photorefractive paraelectric sillenite crystal of 23 symmetry point group, with a strong natural optical activity ρ = 6.3 deg/mm and effective refractive index n = 2.58 [10]. Its static dielectric permeability is 47 with 0.6 cm^{-1} optical absorption. The BTO is the only crystal to show great sensitivity to the red line of the He-Ne laser source in diffusion only recording mechanism[11] In the present experiment it was used a 2WM (two-wave mixing) configuration, appropriate to produce high quality interference patterns in a stable non-active holographic mounting.

In **Figure.1** the experimental setup sketch shows two beams with __= 0.633 _m coming from a He-Ne laser light source with 35 mW nominal power. These beams are expanded and collimated into a 12 mm diameter. The laser light beams, one coming from the mirror **M1**, as an "object", and other from mirror **M2** as the "reference" one, characterize a 2WM(two wave mixing) configuration. Both beams are projected in the (110) plane which is the crystal entrance face at an angle $\theta_B \sim 23^0$. This means, according to the classic Bragg's law, 1,000 lines mm^{-1} spatial frequency.

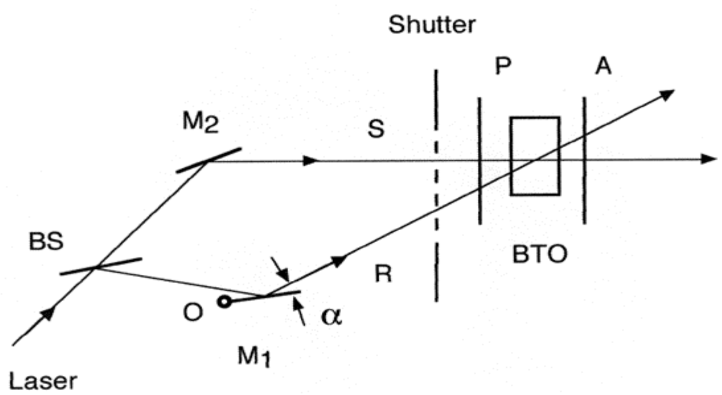

Figure 1: Experimental setup sketch. The mirror **M1** is electronically controlled to produce small rotations around point O.

Recalling **Figure.1**, **BS** is a variable beam splitter, **M1** and **M2** are mirrors, **P** is an entrance polarizer and **A** an analyzer. The mirror **M2** was connected to a computer controlled mechanical support to make desirable rotations (calibrated by an auxiliary experiment) in both clockwise and counterclockwise senses, around the indicated point **O**. Finally, a CCD camera (not shown in the figure), interfaced with an IBM-PC compatible computer, monitors and eventually registers the dynamic hologram. The captured image analysis was performed on a Macintosh computer using the public domain *NIH Image* software.

The obtained results are shown in **Figure.2**. A real time double exposure operation is capable to produce the dynamic Moiré-like fringes. Firstly, a real time sinusoidal phase grating is generated. Then, the shutter (**Figure.1**) isolates the photorefrative sample. Opening the shutter to illuminate the BTO crystal with a new sinusoidal interference pattern, according to a same reference axis (normal to incidence plane), produces the second one. In fact, after the first grating has been generated, mirror **M2** is precisely rotated by a very small angle, and immediately after, opening the shutter, the second (third, fourth, fifth…) sinusoidal interference pattern is projected with a slightly different spatial frequency, and the large and typically Moiré shaped fringes appear in the light diffracted beam for each case. The complete physical event approximately occurs at the response time to photorefractive material, when both gratings are generated and superimposed in the sample volume. This response time is ~5 s and the whole process to obtain one Moiré fringe pattern, including the grating generations, shutters operation and CCD image acquisition lasts ~10s.

Results

The main idea is to demonstrate that the large and spaced Moiré shaped fringes(**Figure.2**) appear as a consequence of the dynamic superposition of two or more sinusoidal phase gratings with slightly different pitches superimposed in the BTO crystal sample volume. The fringes appear in the diffracted light due to the photorefractive index modulation predicted by both Eq.(6) and Eq.(9). It can be observed that the Eq(9) representing the index modulation can be identifyed as a Fourier serie, that is, a Fourier syntesis of the Moiré-like patterns with a "squared" profile tendency when the number of the superinposed gratings are increased in the experimental process.

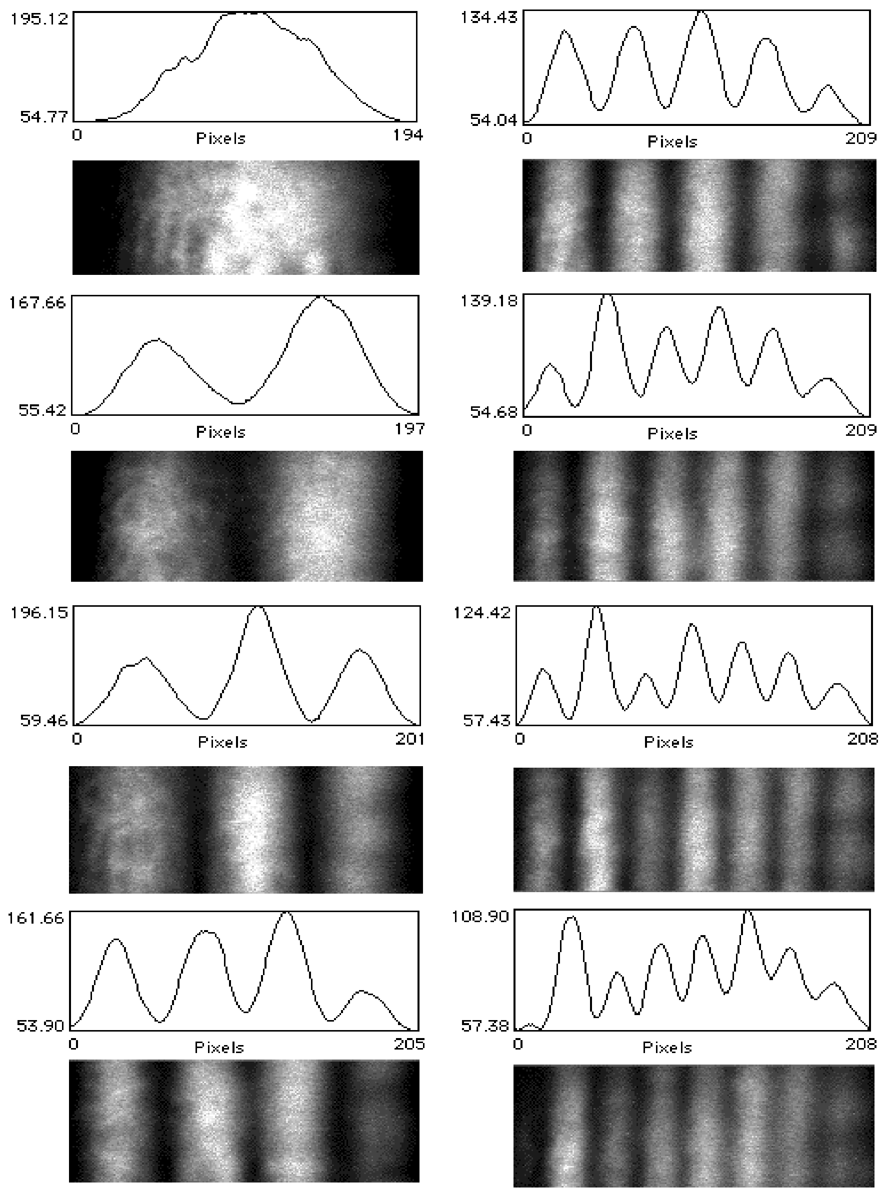

Figure 2 Dynamic Moiré patterns produced by two sinusoidal phase grating superposition. According to Eq(9), in the left column, up to down the j = 1,2,3 and 4. In the right column, up to down the j = 5,6,7 and 8.

Conclusion

Recently we have demonstrated that dynamic Moiré patterns can be produced by rotation [6] or superpositon [12,13] of two photorefractive sinusoidal phase gratings. In the present work the large and spaced Moiré shaped fringes appear as consequence of the dynamic superposition of many sinusoidal phase gratings superimposed in the BTO crystal sample volume. That is, these fringes are due to the resulting photorefractive index modulation predicted by the simple developed physical model. According to this model, the observed Moiré-like fringe patterns are produced as a **Fourier syntesis** phenomenon, treated exactly like the interference of two or more near frequency traveling waves, but due to the dynamic superposition of more than two induced sinusoidal refractive index modulations or phase gratings in the BTO crystal sample volume.

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