# A new approach to simulate modulated speckle patterns with a generalized pupil function

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**Abstract**

A general formalism to calculate same one modulated speckle pattern, which is based in a generalized pupil function and use the random walk model to allow a further analysis of the phenomenon is developed. We demonstrate that the usually geometrical image approximation fails to describe the modulated speckle patterns, for this a new approach based in a local phase approximation is used. The validity of the approximations employed is verified by comparing the simulations with experimental results.

The speckle is generated as a result of the multiple interference scattered from a diffusing object illuminated by coherent light [1]. A variety of speckle applications, from static to dynamic displacements [2], strain measurements [3], phase measurements [4] and temporal speckle analysis to determine biological behavior [5, 6], have been demonstrated.

In all applications a continuous model for the speckle field is used restricting, thereby, the results to mean values. We show that the random walk model could help to understand and to a further analysis of the phenomenon based on a point by point approach. This model allows to describe a physical magnitude as a finite sum of random phasors and is usually utilized to describe several effects involved in the propagation of electromagnetic waves [7]. The random walk model has been of particular interest in the case of interacting and non-interacting particle systems [8, 9]. Besides, in the case of scattering by continuous media such as diffusers [10], rough surfaces [11] and extended regions of turbulence [12], the model represents an adequate tool to describe the phenomenon. Also, it could be employed to describe a digitalized image [13]. Recently, it was defined the "sampled speckle" that appears as a consequence of a digitalized process [14]. Note that the sampling process of an scene into a number of pixels can be consider as a random distribution of intensities. Then, the diffuser that generate the speckle field can be assumed as an ensemble of scattering elements with any distribution of shape, position and phase.

In particular, we propose to revise the analysis of the modulated speckle pattern that appears in several metrological application [2, 4]. In this sense we develop a general formalism to calculate the modulated speckle pattern based in a generalized pupil function. The use of the random walk model allows us to simulate the modulated speckle pattern. Whit the resulting simulation we find that the geometrical image approximation fails to describe thoroughly the modulated speckle pattern. Then, we give a new approach to analyze this phenomenon.

Let us consider the experimental set-up of Figure 1. A laser beam of wavelength $\lambda$ impinges on the diffuser surface ($x-y$ plane). An image of the diffuser is formed in the $X-Y$ plane by use of a lens L of focal length $f$. A pupil mask P with several apertures $a_h$ (where $h = 1, 2...n$) is located immediately in front of the lens. There is not overlapping between two different apertures belonging to the same pupil. Note that, the line that joints a determined aperture and the optical axis of the imaging system forms an angle $\alpha_h$ with the horizontal axis ($u$ -axis). Also, the center of the aperture $a_h$ is located at a distance $d_h$ from the optical axis of the system. $Z_0$ and $Z_C$ denote the distance from the diffuser to the lens and from the lens to the image plane, respectively.

Let us introduce an experimental result to motivate the theoretical approach to be presented in this work. In Figure 2 a speckle pattern obtained in the $X-Y$ plane with the experimental arrangement of Figure 1 is shown. The image is generated by use of a double aperture pupil with circular holes of diameter $D = 4.8mm$ and separated a distance $d = 9,2mm$ ($\alpha_1 = \pi/2$, $\alpha_2 = -\pi/2$). The parameters employed are $\lambda = 514nm$, $Z_0 = 138mm$ and $Z_C = 382mm$. The experimental image is recorded by use

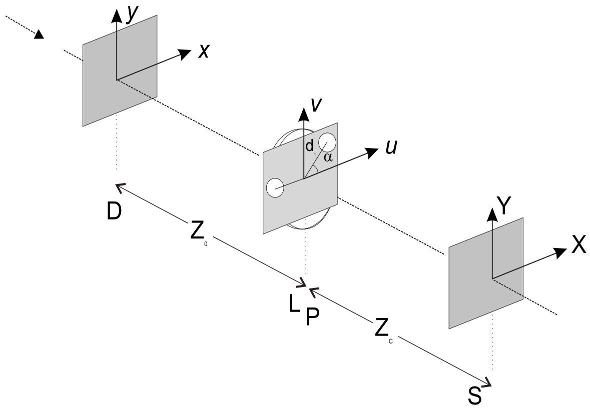


Figure 1: Experimental set-up used to obtain modulated speckle patterns (D: diffuser; L: imaging lens; P: pupil mask; S: image plane).

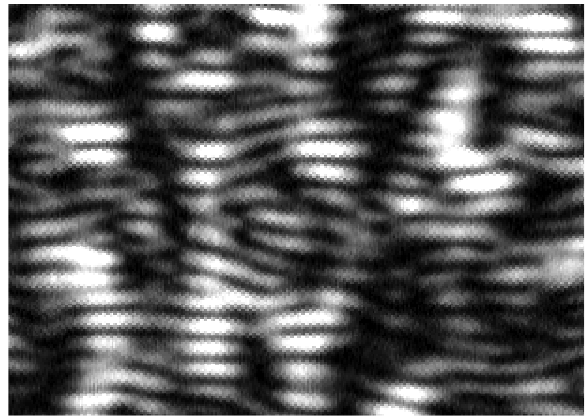

Figure 2: Experimental modulated speckle pattern.

of a CCD camera, with a zoom-microscope providing a lateral magnification of 4.5x and focused on the observation $X-Y$ plane. The image is applied to a region whose actual dimensions are $0.63mm \times 0.43mm$.

Let us analyze theoretically the amplitude distribution in the image plane. The speckle image field in the $X-Y$ plane is obtained by use of the Rayleigh-Sommerfeld expression (see appendix ref. 2). Considering a generalized pupil function consisting of a sum over all the apertures, that is $P(u,v) = \sum_{h=1}^{n} a_h(u - d_h \cos\alpha_h, v - d_h \sin\alpha_h)$. By introducing a variable change $u_h = u - d_h \cos\alpha_h$, $v_h = v - d_h \sin\alpha_h$ for each aperture. Taking the well known paraxial approximations $Z_0, Z_C \gg d_h, u_h, v_h$. And write down the speckle field using the random walk model, that is $U_0(x,y) = \sum_{q=1}^{r} U_q(x,y) \exp\{j\phi_q\}$, where $r$ is the number of speckles and $U_q(x,y)$ and $\phi_q$ are the amplitude and phase, respectively. Then the speckle field results:

$$\begin{aligned} U_i(X,Y) \;=\; & \frac{1}{\lambda^2 Z_0 Z_C} \sum_{h=1}^{n}\sum_{q=1}^{r} \int\!\!\int\!\!\int\!\!\int a_h(u_h,v_h) \exp\left\{\frac{-jk}{2Z_C}\left[(d_h \sin\alpha_h - Y)^2 + (d_h\cos\alpha_h - X)^2\right]\right\} \\ & \times \exp\left\{\frac{-jk}{2Z_0}\left[(d_h\sin\alpha_h - y)^2 + (d_h\cos\alpha_h - x)^2\right]\right\} \exp\left\{jk\left[\frac{d_h^2}{2f} - (Z_0 + Z_C)\right]\right\} \\ & \times \exp\left\{jk\left[u_h\left(\frac{x}{Z_0} + \frac{X}{Z_C}\right) + v_h\left(\frac{y}{Z_0} + \frac{Y}{Z_C}\right)\right]\right\} U_q(x,y)\exp\{j\phi_q\}\, dxdydu_hdv_h. \qquad (1) \end{aligned}$$

where $k = 2\pi/\lambda$ and $j = \sqrt{-1}$. Note that Eq. (1) show explicitly the coordinate system variable associated to each aperture. The discrete field components $U_q(x,y)\exp\{j\phi_q\}$ are small spots with random amplitude, phase and position $(x_q, y_q)$.

At this point it is important to analyze the square phase terms in Eq. (1). The term $\exp\{-jk[(d_h\sin\alpha_h - Y)^2 + (d_h\cos\alpha_h - X)^2]/2Z_C\}$ is relevant because it depends on the summation index. In previous paper, when a single aperture pupil is considered, this term goes out of the integral and vanishes if the intensity evaluation is done. The second square term to be analyzed is: $\exp\{-jk[(d_h\sin\alpha_h - y)^2 + (d_h\cos\alpha_h - x)^2]/2Z_0\}$. In previous work [15], for instance, this term is disregarded because a geometrical image approximation is done. In this approach, the light amplitude at coordinates $(X,Y)$ must consist of contributions belonging to a tiny region of the object space, centered in the ideal geometrical object point. If within that tiny region the argument of $\exp\left\{jk(x^2+y^2)/2Z_0\right\}$ changes by no more than a fraction of a radian, then the approximation $x \cong X/M$ and $y \cong Y/M$, where $M$ is the image magnification, is used.

With the approximations concerning the second square term [15] detailed above and neglecting the phase terms that go out of the integrals and sums, the amplitude field results:

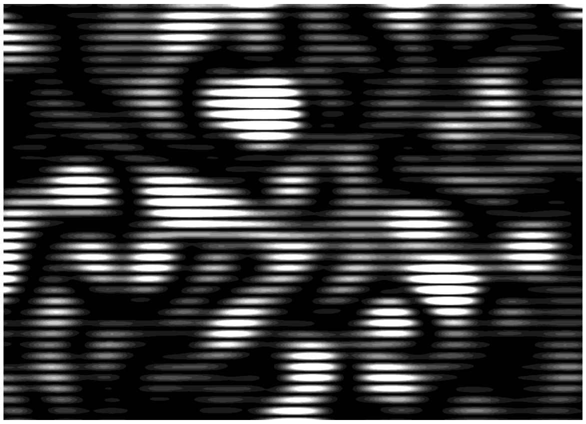

Figure 3: Theoretically modulated speckle pattern obtained by use of the geometrical image approximation.

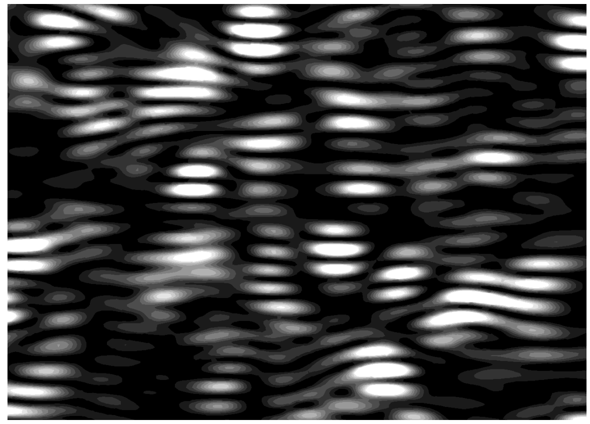

Figure 4: Theoretically modulated speckle pattern obtained by use of the local phase approximation.

$$U_i(X,Y) = \frac{1}{\lambda^2 Z_0 Z_C} \sum_{q=1}^{r} \sum_{h=1}^{n} U_{qh}(X,Y) \exp\left\{j\left(\phi_q + \phi_h(X,Y)\right)\right\}. \tag{2}$$

$$\phi_h(X,Y) = \frac{-2kd_h}{Z_C}(Y \sin\alpha_h + X \cos\alpha_h). \tag{3}$$

$$U_{qh}(X,Y) = \int\int dxdy U_q(x,y) A_h(x,y;X,Y). \tag{4}$$

$$A_h(x,y;X,Y) = \int\int a_h(u_h,v_h) \exp\left\{jk\left[u_h(\frac{x}{Z_0} + \frac{Y}{Z_C}) + y_h(\frac{y}{Z_0} + \frac{Y}{Z_C})\right]\right\} du_h dv_h. \tag{5}$$

To compare with the experimental results, the same double aperture pupil system as in Figure 2 is considered. In this case, the intensity results:

$$I_i(X,Y) = \frac{2}{(\lambda^2 Z_0 Z_C)^2}\left(1 + \cos(\frac{2kdY}{Z_C})\right)\left[\sum_{q=1}^{r} U_q^2 + 2\sum_{q=1}^{r-1}\sum_{s>q}^{r} U_q U_s \cos(\phi_q - \phi_s)\right] \tag{6}$$

A simple geometrical analysis shows that the spatial frequency for a double aperture system is $|\vec{K}| = kd/Zc$. However, from Eq. (6) the spatial frequency is $|\vec{K'}| = 2kd/Z_C$. This discrepancies are shown in Figure 3 (compare with Figure 2.).

Also, it is observed that the fringe modulation phase in the theoretical pattern does not depend on the speckle position. This behavior does not coincide with the experimental result.

Let us reinterpret the approximations in the second square term $\exp\{-jk[(d_h \sin\alpha_h - y)^2 + (d_h \cos\alpha_h - x)^2]/2Z_0\}$ to avoid these discrepancies.

The speckle field in Eq. (1) implies to evaluate the integration in each spot. Due to its small size, it is valid to fix a local phase value by assuming a low phase variation in it. This local phase implies to replace $(x,y)$ by $(x_q,y_q)$ in the mentioned square term.

The image speckle field generated by a multiple aperture pupil, under the conditions detailed above and neglecting phase terms that go out of the integrals and sums results:

$$U_i(X,Y) = \frac{1}{\lambda^2 Z_0 Z_C} \sum_{q=1}^{r} \sum_{h=1}^{n} U_{qh}(X,Y) \exp\left\{j\left[\phi_q + \phi_{qh} + \phi_h(X,Y)\right]\right\}. \tag{7}$$

$$\phi_{qh} = \frac{-kd_h}{Z_0}(x_q \cos\alpha_h + y_q \sin\alpha_h) + \frac{k}{2Z_0}(x_q^2 + y_q^2). \tag{8}$$

$$\phi_h(X,Y) = \frac{-kd_h}{Z_C}(Y \sin\alpha_h + X \cos\alpha_h). \tag{9}$$

and $U_{qh}(X,Y)$ and $A_h$ are given by Eqs. (4) and (5) respectively and remain without change.

Let us evaluate the intensity applied to an identical system as in Figure 2. In this case, the intensity is given by:

$$I_i(X,Y) \propto \sum_{q=1}^{r} U_q^2 \left[1+\cos\left(kd(\frac{y_q}{Z_0}+\frac{Y}{Z_C})\right)\right] + \sum_{q=1}^{r-1}\sum_{s>q}^{r} U_q U_s \cos\left(\phi_q-\phi_s+\frac{k}{2Z_0}(y_q^2-y_s^2+x_q^2-x_s^2)\right) \times \left[\cos\left(\frac{kd}{2Z_0}(y_q-y_s)\right)+\cos\left(kd(\frac{y_q+y_s}{2Z_0}+\frac{Y}{Z_C})\right)\right]. \quad (10)$$

This result is depicted in Figure 4. Note that the fringe frequency coincides in Figures 2 and 4 and also a local phase behavior as in the experimental case is observed.

To summarize, we demonstrate that by use of the random walk model and with a new approach for the optical model, it is possible to simulate modulated speckle patterns whose behavior coincide with the experimental results. The analysis is general and could be applied to more complex aperture pupils.

This approach that gives a more precise description of the speckle local behavior could be useful to control the diffuser features, for instance, its statistic (fully developed or gaussian), surface roughness, etc. Besides, concerning the modulating fringes in multiple aperture system, it could be advantageously employed to control interference phase measurement techniques [16].

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