Output coupling of fermionic trapped atoms

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Abstract

In this work we analyze the output coupling from a magneticaly trapped state into a free particle state of a fermionic species. Within the formalism of Laplace transforms, we develop general expressions describing the outcoming beam, its first and second order coherences. While the results are valid for a general trap potential coupled by a Dirac-delta function at the origin, here, we discuss the 1-D Box potential case. The output beam energy spectra show different behaviour depending of the coupling strength. For weak coupling, the spectrum of the beam reveals the trap level's structure with an envelop whose amplitude grows with the energy. For strong coupling, the spectrum is shifted away from the trap levels because of an effective interaction between them. Contrarily to the weak coupling regime, the first output mode is preferentially populated. The second order correlation function exhibits the expected property of anti–correlation. A complete physical understanding of the results is still underway.

The experimental demonstration of Bose-Einstein Condensation [1] was followed by a demonstration of a coherent beam of bosonic atoms, known as atom laser [2]. Since then, much interest was devoted to the theoretical studies of these new devices[3, 4]. Recently, the interest has been migrating towards ultracold degenerate samples of fermionic atoms and their use for atomic beams[5, 6, 7].

Here, we use the Laplace transform formalism [8, 4] to develop a dynamical theory of the output coupling of fermionic atoms. We obtain the main equations for a general one-dimensional trap and solve for the one-dimensional box case, as a suitable example. We consider one species of fermionic atoms with two different internal states. The first internal state (spin up) is trapped while the second one (spin down) is insensitive to the potential. The coupling, which induces transitions between the internal states also couple the center-of-mass, translational states. The shape of the coupling potential is constrained to spatial Dirac-delta function. This simple model leads to analytical solutions for the Laplace Transforms of the Equations of Motion.

The Hamiltonian for this system is written as

$$H = \sum_n \hbar\omega_n a_n^\dagger a_n + \int d\xi \hbar\omega(k) b_k^\dagger b_k + i\hbar\bar{\lambda} \sum_n \int d\xi \psi_k^*(0)\varphi_n(0) b_k^\dagger a_n + H.c. \tag{1}$$

where a_n is the operator that annihilates an atom in the level n of the trap, while b_k is the annihilation operator of the free particle with momentum k. All these operators obey the fermionic anti-commutation relations. The spatial shape of the coupling is given by $\lambda(x) = \bar{\lambda}\delta(x)$.

The Heisenberg Equations of motion for the operators become

$$\dot{a}_n(t) = -i\omega_n a_n(t) - \bar{\lambda}\varphi_n^*(t) \int dk \psi_k(0) b_k(t)\,, \tag{2}$$

$$\dot{b}_k(t) = -i\omega_k b_k(t) + \bar{\lambda}\psi_k^*(0) \sum_n \phi_n(t) a_n(t)\,, \tag{3}$$

where $\psi_k(r) = e^{-ikr}/\sqrt{2\pi}$. This set of equations can be solved by the formalism of Laplace Transform, the details of which can be found in references[9, 10].

We assume that all the atoms are initially trapped, so that $b_k(0) = 0$. The solutions of the Laplace transformed operators are given by:

$$\tilde{a}_n(\varepsilon) = \frac{a_n(0)}{\varepsilon + i\omega_n} - \bar{\lambda}^2 \frac{\varphi_n^*(0)}{\varepsilon + i\omega_n} \frac{I(\varepsilon)}{1 + \bar{\lambda}^2 I(\varepsilon) F(\varepsilon)} \sum_j \frac{\varphi_j(0)}{\varepsilon + i\omega_j} \tilde{a}_j(0)\,, \tag{4}$$

$$\tilde{b}_k(\varepsilon) = \bar{\lambda} \frac{\psi_k^*(0)}{\varepsilon + i\omega_k} \times \frac{1}{1 + \bar{\lambda}^2 I(\varepsilon) F(\varepsilon)} \sum_n \frac{\varphi_n(0)}{\varepsilon + i\omega_n} \tilde{a}_n(0)\,. \tag{5}$$

The functions $I(\varepsilon)$ and $F(\varepsilon)$ depend respectively on the potentials outside and inside the trap. These functions are given, in general, by the equations

$$I(\varepsilon) = \int d\xi \frac{|\psi_\xi(0)|^2}{\varepsilon + i\omega_k}, \tag{6}$$

$$F(\varepsilon) = \sum_n \frac{|\varphi_n(0)|^2}{\varepsilon + i\omega_n}. \tag{7}$$

Others interesting quantities are the correlation functions of first and second orders of the outcoming beam. They are based on the field operators outside the trap, defined as $\hat{\Psi}_k(x) = \int dk \psi_k(x) b_k$. We can define the normalized first order correlation function as

$$g_1(r, r', t) = \frac{\langle \hat{\Psi}^\dagger(r', t)\hat{\Psi}(r, t)\rangle}{\sqrt{\langle \hat{\Psi}^\dagger(r', t)\hat{\Psi}(r', t)\rangle \langle \hat{\Psi}^\dagger(r, t)\hat{\Psi}(r, t)\rangle}}. \tag{8}$$

Based on this definition, we can write the normalized second order correlation function. This quantity for fermions has the fundamental property of anti-correlation. It can be written as

$$g_2(r, r', t) = 1 - |g_1(r, r', t)|^2. \tag{9}$$

Now, we apply these results to the particular case of a one-dimensional box potential. In ref. [9, 10] we also develop the Harmonic Oscillator case. We take a box with length L, whose energy levels and wavefunctions are given by:

$$\hbar\omega_n = \frac{\hbar\pi^2}{2ML^2} n^2, \tag{10}$$

$$\varphi_n(r) = \sqrt{\frac{2}{L}} \cos(k_n r), \tag{11}$$

where $k_n = (2n + 1)\pi/L$, and we have ignored the odd wavefunctions as these are not coupled at the origin. We then obtain for $I(\varepsilon)$ and $F(\varepsilon)$

$$I(\varepsilon) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{dk}{\varepsilon + i\omega_k} = \frac{1}{2\pi} \sqrt{\frac{2m}{\hbar}} \frac{\pi}{\sqrt{i\varepsilon}}, \tag{12}$$

$$F(\varepsilon) = \frac{2}{L} \sum_{n(\text{odd})} \frac{1}{\varepsilon + i\omega_n} = -\frac{1}{4} \sqrt{\frac{2imL^2}{\hbar\pi^2}} \frac{2\pi}{L\sqrt{\varepsilon}} \tan\left(\pi \sqrt{\frac{imL^2\varepsilon}{2\hbar\pi^2}}\right). \tag{13}$$

The expression for the annihilation operator outside the trap becomes

$$\tilde{b}_k(z) = \frac{\delta\sqrt{L}}{\sqrt{\pi}} \frac{z}{z + i(kL)^2} \frac{\cos\left(\frac{\sqrt{iz}}{2}\right)}{z\cos\left(\frac{\sqrt{iz}}{2}\right) - \delta^2 \sin\left(\frac{\sqrt{iz}}{2}\right)} \sum_{n=1,3,5,\dots} \frac{a_n(0)}{z + i\pi^2 n^2}, \tag{14}$$

where we have defined the scaled Laplace variable $z = \pi^2\varepsilon/\omega_1$ and the adimensional coupling constant $\delta = \pi^2\bar{\lambda}/L\omega_1$. The inverse Laplace Transform of the equation Eq. 14 above requires many steps and the interested reader is referred to the references[9, 10]. The result for the annihilation operator in the long–time behaviour is given by

$$b_k(\tau \to \infty) = \sum_{n=1(\text{odd})}^{+\infty} \frac{\delta\sqrt{L}}{\sqrt{\pi}} \Big[\frac{(kL)^2}{(kL)^2 - \pi^2 n^2} \times \frac{\cos(|kL/2|) e^{-ik^2L^2\tau}}{-i(kL)^2 \cos(|kL/2|) - \delta^2 \sin(|kL/2|)}$$
$$+ \frac{\bar{z}_0}{\bar{z}_0 + ik^2L^2} \times \frac{1}{1 - \frac{\sqrt{i}\bar{z}_0^{(3/2)}}{4\delta^2} - \frac{\delta^2\sqrt{i}}{4\sqrt{\bar{z}_0}}} \times \frac{e^{\bar{z}_0\tau}}{\bar{z}_0 + i\pi^2 n^2} \Big] a_n(0), \tag{15}$$

where the scaled time $\tau = \omega_1 t/\pi^2$ and $\bar{z}_0$ is the only non–trivial pole of eq. (14) in the Imaginary axis. It can be shown that only the first term in Eq. 15, equivalent to the $z = -i(kL)^2$ pole, significantly contributes to the total value of the expression [4].

The spectra are obtained by numerical evaluation of $\langle b_k^\dagger b_k \rangle$ and are shown in Fig. 1 for different coupling constants. For $\delta = 0.1$ we clearly observe the ressonance peaks from the original discrete states of the trap. It is important to notice that the higher trap quantum states contribute more effectively to the bidirectional output beam. For $\delta = 100$ the peaks are shifted towwards new values of energy, caracterizing "dressed" states of the trap levels in contact with the non-trapped levels.The behaviour with energy (kL) shows a kind of mode selection, with the first peak having a greater intensity than the others. The physical causes and consequences of these behaviours are currently under discussion.

In Fig. 2 we plot the second order correlation function as a function of the scaled position $R = r/L$. Although, in both cases we observe the anti–correlation between diferent positions of the atoms, this property is more pronounced in the case of strong coupling.

In conclusion, we have developped a dynamical model for the output coupling of fermions from a trap. Our strategy for this problem was to deal with a simple model, that it was possible to handle analyticaly, but sufficiently rich to demonstrate interesting features of the system. Remarkable effects are theoretically observed and the complete physical discussion is underway. It is interesting to notice that our model also allows us to deal with more realistic situations, such as the presence of a gravitational field.

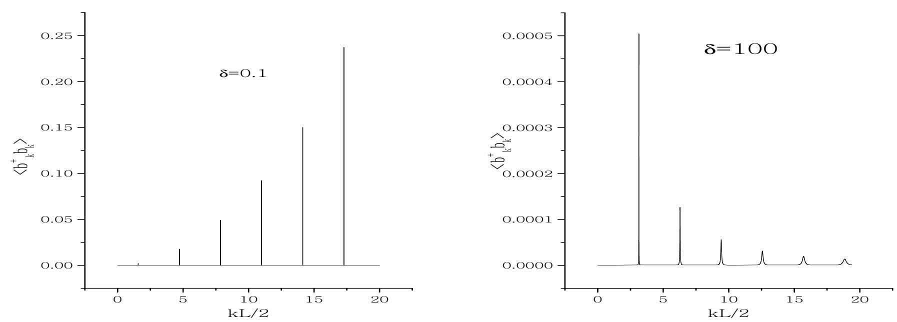

Figure 1: Spectrum of the outcoming beam $\langle b_k^\dagger b_k \rangle$ as function of $K = kL/2$ for $\delta = 0.1$ and $\delta = 100$. In the graphics, only the positive momenta are shown.

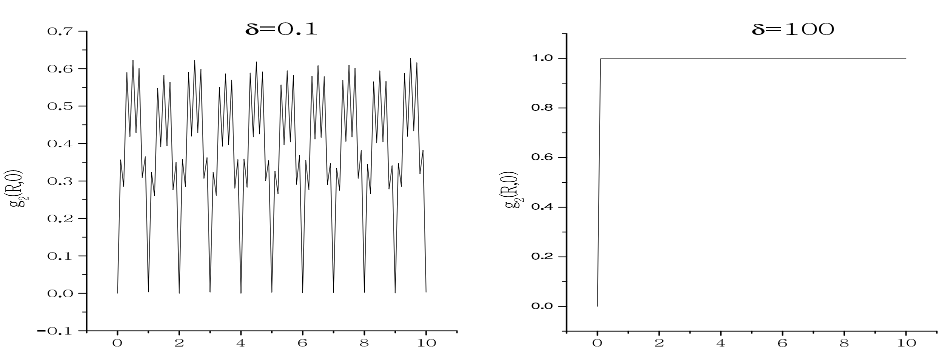

Figure 2: Normalized second order function $g_2(R, 0, \tau)$ for the cases $\delta = 0.1$ and $\delta = 100$ as function of $R = r/L$. We take $\tau = 10$.

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