Using evanescent wave scattering as a theoretical tool to model microscopic rotational motion detectors

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Abstract

We developed an approximate solution for the problem of evanescent wave coupling by resonant light scattering by two non absorbing dielectric cylinders (pump and probe). To minimize the e®ects of multire°ection between the scattering centers a normal incident o®-axis Gaussian beam is used instead of a plane wave. The parameters of the theory are varied in order to reveal some scattering features. Special interest is paid to the beam's waist parameter (w_0) which leads to the plane wave results in an appropriate limit and the angle of rotation (Á$_0$) of one of the cylinders in relation to the other. This scheme is proposed as a possible mechanism for probing molecular dynamics.

Light scattering is a sensitive tool which can be used to determine the geometry and composition of particle systems. More speci¯cally evanescent waves have long been familiar in optics and their properties have been exploited both experimentally and theoretically by several techniques. In the past years, techniques which employ evanescent waves have been used in laboratory to characterize and probe biological systems. It is well known that modeling the scattering from complex media such as biological systems is a di±cult task due to the intricate interaction between light and multiparticle suspension. As a consequence, theorists are de¯ed to elaborate more realistic models in order to give precise coverage of all the detectable e®ects as well as the prediction of new ones. An interesting problem to investigate in biological systems is, for instance, the momentum exerted by a protein during it's folding and unfolding. With the present day detectors such a measurement could be readily made with an appropriate theoretical and experimental approach. In this letter we present a theoretical analysis which results in the proposal of an experimental method that could be used to study single protein physical properties, more speci¯cally, the single protein or particle attached in microsphere execute a rotational motion. A technique to measure microscopic rotational motion was presented in [2], where the two microspheres are used in a microlensing e®ect to measure a small rotations. We consider the problem of light scattering by two parallel non-absorbing dielectric cylinders separated by a distance d. The incident electromagnetic ¯eld is a monochromatic o®-axis normally incident Gaussian beam. A Gaussian beam permits a localized incidence on a certain region of space which minimizes the multire°ection process between the scatterers. In addition, the scattering process occurs such that the incident beam excites an morphology dependent resonance (MDR) in one of the cylinders called `pump' [see Fig. (1)]. The resonance structure is characterized by internal and external caustic regions. The evanescent wave is generated from the external caustic region of the pump [1]. The other cylinder, here denoted as `probe', is not on resonance unless it is positioned into the external caustic region generated by the pump cylinder. In this situation the pump's resonance is coupled into one of the probe's normal modes. The radii and refractive index of pump and probe cylinders are respectively: a_1, a_2, N_1 and N_2: In other to apply the correct boundary conditions on both cylinders the addition theorem for cylindrical functions is used. In this manner it is possible to measure light scattering intensity when the probe (radius a_2) cylinder rotates by an angle (Á$_0$) in relation to the pump (radius a_1) while keeping the value of d ¯xed. Theoretically we calculate the scattered and inner ¯eld related to the probe for various relative angular position. This mapping of the scattering signal and the relative angular position Á$_0$ will give a characteristic functional behavior which will be discussed.

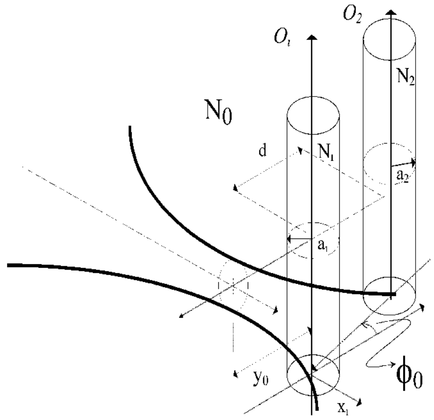

Figure 1: Scattering configuration of two parallel dielectric cylinders.

Let us now consider the problem of Gaussian beam scattering by two parallel non-absorbing dielectric cylinders separated by a distance d. The geometry of the scattering system is shown in Figure I. The pump (probe) cylinder is characterized by radius $a_1(a_2)$ and refractive index $N_1(N_2)$, embedded in a medium of refractive index N_0. Firstly, the pump (probe) cylinder is taken in reference $O_1(O_2)$ and the cylindrical coordinates $(\rho_i;\phi_i;z_i)$ are adopted. The incident electromagnetic field is a monochromatic Gaussian beam. In this manner let us take the incident beam at normal incidence propagating along the x axis with the center of it's focal plane localized at $(x_0;y_0;z_0)$, parallel to the yz plane. The beam's waist and the polarization states are denoted: w_0, μ (TM) and ϵ (TE) respectively. The distance from the center of the beam waist to the origin of the pump cylinder is y_0 and we denote as the impact parameter for resonance excitation.

Using the methods developed in Ref.[1] the fields are described from the view point of a scalar theory. In this manner the total scalar potential for the external region of the two cylinders are:

$$\psi^{\mu}_{Total} = \psi^{\mu}_{inc} + \psi^{\mu}_{scatt\_1} + \psi^{\mu}_{scatt\_2};$$

which are explicitly given for μ-polarization by

$$\psi^{\mu}_{inc} = \int_{-\infty}^{\infty} N_0 dh \sum_{n=-\infty}^{\infty} F_n A^{\mu}_{n(O_1;O_2)} J_n\left(\rho_{1;2} k_0 \sqrt{N_0^2 - h^2}\right) e^{in\phi_{1;2}} e^{ihk_0 z_{1;2}};$$

$$\psi^{\mu;O_j}_{int} = \int_{-\infty}^{\infty} N_j dh \sum_{n=-\infty}^{\infty} F_n a^{\mu}_{n;O_j} J_n\left(\rho_j k_0 \sqrt{N_j^2 - h^2}\right) e^{in\phi_j} e^{ihk_0 z_j};$$

$$\psi^{\mu;O_j}_{scatt} = \int_{-\infty}^{\infty} N_0 dh \sum_{n=-\infty}^{\infty} F_n A^{s;\mu}_{n;O_j} H^{(1)}_n\left(\rho_j k_0 \sqrt{N_0^2 - h^2}\right) e^{in\phi_j} e^{ihk_0 z_j}:$$

Where $F_n = i^{n+1}E_0/k_0$, E_0 is the peak-strength of the electric field, j is the medium index for the refraction indexes $N_{j=1;2}$ and also a reference frame index $(O_{j=1;2})$, J_n, $H^{(1)}_n$ are the Bessel and Hankel (first kind) cylindrical functions respectively and we have chosen to rewrite the dimensionless variable h as $h = k_z/k_0$ and finally $A^{\mu}_{n(O_j)}$ is the beam shape coefficient (BSC) for a Gaussian beam[3].

At this stage, it is possible to find the scalar potentials prescribed above. Consequently, the electromagnetic fields (EM) are promptly obtained. We have already mentioned that the pump cylinder will sustain a (MDR) and the probe will scatter it's evanescent wave. For this reason we choose to take the probe's coordinate system as the reference for our calculations. Thus, we must apply the addition theorem for cylindrical functions to describe the EM fields of the pump taken in the probe's reference system. The translational addition theorem given for $\rho_2 > d$ are:

$$
\begin{aligned}
H_l^{(1)}(\tilde{k}\rho_2)e^{il\phi_2} &= \sum_q H_{q-l}^{(1)}(\tilde{k}d)J_q(\tilde{k}\rho_1)e^{iq\phi_1}e^{i(l-q)\phi_0}; \\
J_l(\tilde{k}\rho_2)e^{il\phi_2} &= \sum_q J_{q-l}(\tilde{k}d)J_q(\tilde{k}\rho_1)e^{iq\phi_1}e^{i(l-q)\phi_0}.
\end{aligned}
$$

where, $\tilde{k} = k_0\sqrt{N_0^2 - h^2}$. Thus, this permits us to apply the boundary conditions on each of the cylinders. Here we specialize the results for the scattered field coefficient of the probe $A^{s;\infty}_{k;O_2}$ given as:

$$
\begin{aligned}
&\sum_{k=-\infty}^{\infty}\left\{\delta_{n;k} - T_n^{O_2}\sum_{q=-\infty}^{\infty} F_qF_kH^{(1)}_{q-k}(w)H^{(1)}_{q-n}(w)T_q^{O_1}e^{i(k+n-2q)\phi_0}\right\}A^{s;\infty}_{k;O_2} \\
&= F_nA^{\infty}_{n;O_2}T_n^{O_2} + T_n^{O_2}\sum_{q=-\infty}^{\infty}(F_q)^2A^{\infty}_{q;O_1}T_q^{O_1}H^{(1)}_{q-n}(w)e^{i(n-q)\phi_0}e^{ik_0h(z_1-z_2)}; \qquad (1)
\end{aligned}
$$

where

$$
T_n^{O_j} = \left(\frac{2i\left(\frac{H_n'(u_j)}{u_jH_n(u_j)} - \frac{\mu_j}{\mu_0}\frac{J_n'(v_j)}{v_jJ_n(v_j)}\right)}{u_j^2\pi[H_n(u_j)]^2\left\{D_1^jD_2^j - (nh)^2\left(\frac{1}{u_j^2} - \frac{1}{v_j^2}\right)^2\right\}} - \frac{J_n(u_j)}{H_n(u_j)}\right) \qquad (2)
$$

$$
u_j = a_jk_0\sqrt{N_0^2 - h^2}; \qquad v_j = a_jk_0\sqrt{N_j^2 - h^2}; \qquad (3)
$$

$$
D_1^j = \left(\frac{H_n'(u_j)}{u_jH_n(u_j)} - \frac{\mu_j}{\mu_0}\frac{J_n'(v_j)}{v_jJ_n(v_j)}\right); \qquad D_2^j = \left(\frac{H_n'(u_j)}{u_jH_n(u_j)} - \frac{\epsilon_j}{\epsilon_0}\frac{J_n'(v_j)}{v_jJ_n(v_j)}\right) \qquad (4)
$$

where the term on the left hand side is translational coefficients explicitly indicating the coupling mechanism between the two scatterers and also the important role played by the separation distance d.

To validate the model some numerical tests have been carried out. The full analysis of the results for $\phi_0 = 0$ and the plane wave limit for $w_0 \gg \lambda_0$ are presented in Ref.[1]. One of the tests consists in obtaining single cylinder results when the refractive index of other cylinder is taken to be equal to the surrounding medium. This test is done with $a_1 = a_2 = a$ and $x_0 = z_0 = 0$. In the context of Van de Hulst's principle of localization it is known that in a wave front of a plane wave each multipole order has an associated impact parameter [4]. In the plane wave spectrum approach where a Gaussian beam is decomposed into a spectrum of plane waves propagating in different directions one finds that each multipole that composes the spectrum has a different field amplitude. The peak of this amplitude is exactly positioned on the main plane wave contribution which for normal incidence is $k_z = 0$. In this manner for a certain multipole order m and a size parameter β; resonance excitation happens when the center of the focal plane of the incident beam coincides with the impact parameter explicitly given by the localization principle which can be stated as

$$y_0/a = m/\beta;$$

where we have normalized in values of the radius a. The values of y_0 corresponds to the center of the beam's focal plane, thus it is the impact parameter. The test is done by taking the two cylinder at $d = 0$ and making one of them "invisible". We then plot the square modulus of the scattered field coefficient calculated via Eq. (1) for each one of the cylinders at a time ($|A^{\infty}_{scatt}|^2$) taking a range of values of the beam's focal plane position y_0 along the y axis. It was verified that the peak value of each of the cylinder's associated field coefficient $|A^{\infty}_{scatt}|^2$ is located at impact parameters which coincide exactly to the ones estimated by the localization principle. Specifically, for $n = 23$; $\beta = 17.4983$; the impact parameters for exciting a resonance at the pump is $y_1/a = 1.31441$ and at the probe is $y_2/a = -0.6855$. These results are in accordance with the scattering of a Gaussian beam by a single cylinder[3]. With this facts in mind we seek for the same test now using values in which $\phi_0 \neq 0$. Taking the same scattering configuration as in the first test the scattering coefficients are calculated as functions of ϕ_0 only. In this manner the localization principle is no longer given by $y_o/a = m/\beta$. It must take into account the new variable ϕ_0.

Taking the probe cylinder as the rotating reference frame we ¯nd that to have an e±cient resonance coupling between the beam and one of the internal modes the relative angular position must be:

$$\phi_0 = \arccos(k_0 d/2m)$$

which gives for $d = 2a, y_o = a = m/\beta$ the value of $\phi_0 \approx 40^o$. In Figure (2) we have the plots of the scattering coe±cients for rotating one of the reference frames while the other is kept ¯x and \invisible". It can be veri¯ed that the probe has a maximum at the angle of approximately 40^o just as the prediction stated above.

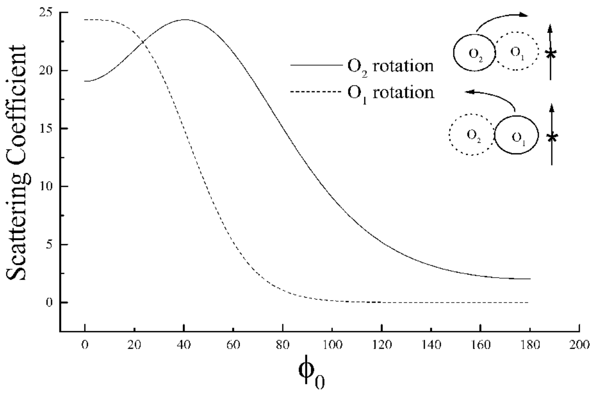

Figure 2: Plot of the Scattering Coe±cients against the relative angular position. The inset shows the scheme for rotation. The arrow with a star indicates the position where the Gaussian beam is focused at.

In conclusion, in this work we have theoretically investigated the problem of resonant light scattering of an normal incidence o®-axis Gaussian beam by two dielectric cylinders separated by a distance d and rotated by ϕ_0: Here we are interested in studying resonance excitations in only one of the scattering centers when illuminated by the incident beam. In this manner a localized o®-axis beam was adopted in order to avoid the strong radiation feedback between the pump and the probe cylinders. Thus the choice of an o®-axis Gaussian beam permits that only the pump cylinder be the resonant scattering center while the probe cylinder at certain regions behave as a simple non-resonant scattering center. Now we are developing the numerical algorithms to investigate how does the intensity of the scattered light of both cylinders behaves as a function of the rotation angle ϕ_0 and in situations such as $a_1 \gg a_2$.

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