Simulation of a coherent image amplifier by using finite differences

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Abstract

This paper presents a numerical model that intends to study theoretically the effects of the saturation of a high gain, non-homogeneous broadened active medium on the process of image formation in an optic system. This is a ab initio solution of the paraxial wave equation by using the Cranck-Nicholson finite difference method implemented in the MathCAD 2001 software.

Introduction

The first experimental results of an optical system filled with a high gain active medium was reported in [1]. This work proposes to analyze this problem numerically, by solving the wave equation, considering diffraction effects and a high gain saturable active medium inside a microscope. The wave propagation is solved by using the finite differences [2,3], more specifically the Crank-Nilcolson method, implemented in the software MathCAD 2001.

First, considering an optical system without active medium, it was compared the results obtained by calculating the wave propagation both by using the Fourier optics method [4] and with the so called Crank-Nilcolson difference equations method, as a validation for the second one. Later, it was considered an constant gain and at last a gain dependent on the radius was taken into account. The results are preliminary yet, but it is already possible to conclude that there is a considerable effect of the non uniform gain on the image formation in the microscope considered here.

The model

The optical system studied here is a projection microscope, considered free of aberration, with a total magnification of 100×, made of a 10× eyepiece and NA = 0,35; the lenses are put 1 m apart and the interlenses space is filled with an active medium with the characteristics of a copper laser (active medium length 1 m, diameter 25 mm, small signal gain 2 mm^{-1}). The distance between object and the real image is 2 m. First it is calculated the image formation by using the Fourier optics and it is supposed that the object is illuminated by coherent light (Cu laser). The object electrical field $u_{obj}(x_i)$ is propagated until the eyepiece, it is phase modulated by this first lens, then it propagate until the second lens (without gain), is phase modulated by the second lens and finally propagate until the image plane. First, all the propagation is calculated by solving the Fresnel-Kirchhoff

$$u_i(x_i) = u_{obj}(x_i) * f(x_i) \tag{1}$$

$$f(x_i) = \frac{1}{i.\lambda.(z-z_0)}.\exp\left[\ i.k\left((z-z_0)+\frac{(x_i)^2}{2.(z-z_0)}\right)\right] \tag{2}$$

by using fast Fourier transform procedures[4]. In the equations above $u_i(x_i)$ is the electric field , at a distance $(z-z_0)$ from the object and $f(x_i)$ is the point-source transfer. The phase modulation due to a lens is given by

$$f(x) = \exp\left[\ i.k\left(\frac{(x)^2}{2.F}\right)\ \right]\ , \tag{3}$$

where F is the lens focal length.

Active medium

The inter-lenses space is filled with a high gain amplifying medium, with a field gain $\alpha(x,z,t)$ dependent on position and time. The influence of an active medium in an amplifier can be described by a partial differential equation that simulates the propagation of a monochromatic and linearly polarized beam in the *z* direction. The paraxial approximation for the wave propagation equation, considering gains[2], is

$$\frac{\partial u}{\partial z} = -\frac{i}{2.k}.\frac{\partial u}{\partial x} + \alpha.u \qquad (4)$$

where $u = u(x,z,t)$ is a scalar function that describes the amplitude of the electromagnetic field,

Crank-Nicholson method

The Crank-Nicholson method is a finite difference approach that, after the discretization of the integration region with Δz in the longitudinal direction and Δx in the transverse one, transforms the differential equation (4) in the finite difference equation

$$-r.U_{i+1,j} + (2+2.r).U_{i,j} - r.U_{i-1,j} = r.U_{i+1,j} + (2-2.r).U_{i,j} + r.U_{i-1,j} + \alpha_{i,j}.U_{i,j} \quad . \qquad (5)$$

Eq. (05) gives rise to a three-diagonal matrix whose solution can be obtained by using the Thomas algorithm, met in the literature[3] .

Results

Fig. 1 shows the test object field distribution, chosen asymmetric in order to allow the observation of inverse image formation. Fig. 2 shows the image, calculated by using Fourier optics, inverse and magnified 100 times, as expected. It can also be observed the diffraction effects due to the "hard edge effect".

Fig. 3 shows the same image, calculated by using the finite difference method, described above. It is observed an excellent agreement between this result and that one shown in Fig. 2, giving confidence that this later method gives good results in image formation calculation.

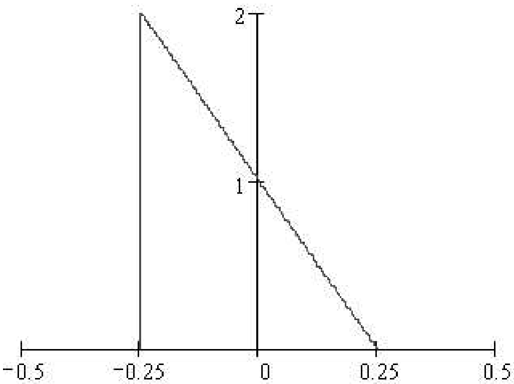

Figure 1: Object (Amplitude $\times$ Transverse dimension [mm])

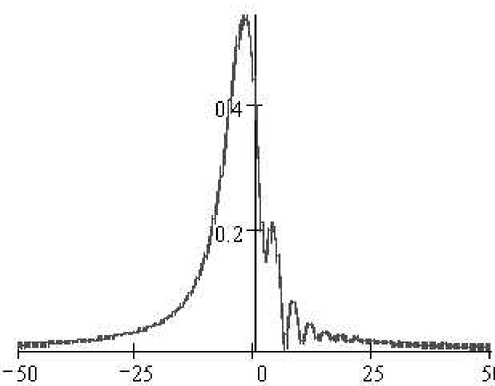

Figure 2: Image, calculated by using the Fourier optics (Amplitude $\times$ Transverse dimension [mm])

Fig. 4 shows the result for a system filled with a constant gain active medium. Since the gain is homogeneous and non saturable, the relative profile must be the same as in the previous case, only the amplitude must augment due to gain. Fig. 5 shows the same situation but with a gain that varies radially, as a parabola that vanishes for $|x| = 8$ mm.. It can be observed a shrinkage of the image due to this gain profile.

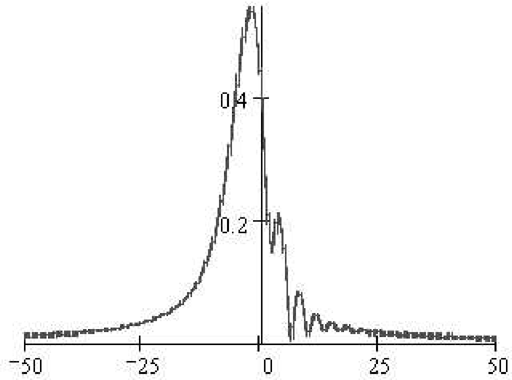

Figure 3: Image, with the propagation between lenses calculated by using finite differences method and null gain .

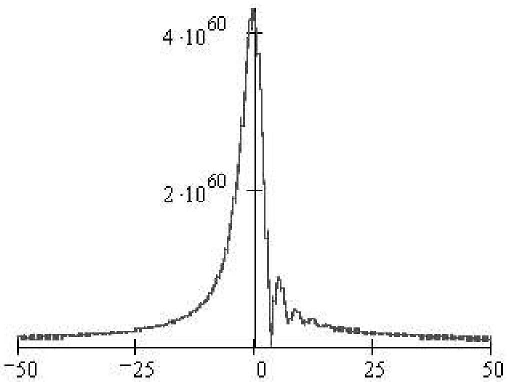

Figure 4: Image, with the propagation between lenses calculated by using finite differences method and constant gain $\alpha = 2\ mm^{-1}$.

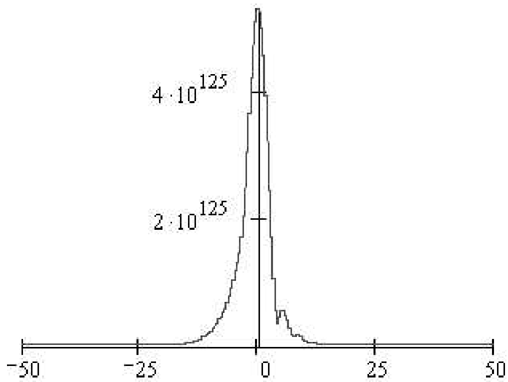

Figure 5: Image, with the propagation between lenses calculated by using finite differences method and a gain with peak value of $\alpha = 2\ mm^{-1}$ and a parabolic gain. .

Conclusions

The results obtained with an empty optical system, calculated by using the finite differences method, agree very well with the Fourier optics predictions, being understood as a validation for the *ab initio* method. The calculation with an uniform gain only confirmed the expected increase in brightness, while the calculation with radius dependent gain indicated deformations in the image.
The next step for this work is to consider gain saturation, two dimension modelling and new optical configurations.

This kind of image amplifier finds applications in high resolution microscopy, spectroscopy and in military applications.

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