Electromagnetic field absorbed in a layers system
Helena Libardi¹ Hans-Peter Grieiesen²

1-Departamento de Física e Química - CCET - Universidade de Caxias do Sul - RS
2-Instituto de Física - Universidade Federal do Rio Grande do Sul - RS
hlibardi@terra.com.br

Abstract
The system studied considers three-layer film system with losses and uses a prism as optical coupler. A laser beam enters the sample with an angle larger than the critical angle at the prism/air interface and the reflected light is calculated. For the calculations the wavelength of the He-Ne laser at 632.8 nm is considered. When the light couples to the optical modes, a minimum intensity for the reflected signal is observed. We calculate the radiative flux density in the different layers and also the energy density dissipated in these layers for a given angle of incidence. Both quantities give inside about the nature and location of the losses within the film structure, which is of special interest in thin film applications for sensors and nonlinear optical device applications.

Introduction
Depending on the index of refraction and the thicknesses of the layers, it is possible to couple light in p-polarization to a surface plasmon (SP), or to the guided modes, in either s- or p-polarizations. Surface plasmons propagate along the metal/dielectric interface. The guided modes are mainly propagated within the dielectric layer. We include a small imaginary part in the dielectric constant, necessary to allow for the study of the dissipation in these dielectric layers, since otherwise there would be no absorbed power, although there could be significant field enhancement. The component of the Poynting vector parallel to interface, the x-axis, and normalized to the total incident flux is calculated and compared with calculations of the normalized power density absorbed in this layer system.

Theory
When light becomes coupled resonantly to an optical mode of a multiple layer system, the electromagnetic field inside the layer may increase considerably. It is straightforward to calculate the electromagnetic fields using Maxwell’s equations with the appropriate boundary conditions and solve for the tangential and normal field components of \( \mathbf{E} \) and \( \mathbf{H} \) at each interface. We are specially interested in the regime of incident angles beyond the critical angle for which normally total internal reflection is observed in both polarizations. For certain angles of incidence light will couple to waveguide modes, being followed by the occurrence of pronounced reflectance minima. Experimentally the angular reflectance spectrum is known as the ATR technique, standing for Attenuated Total Reflection (ATR). Latter has become of special importance in the design and application of optical thin films for sensors and devices.

The reflection coefficient of the field amplitude of the system is calculated, considering a N layer’s system, as:

\[
r = C_{21ij}^{C}
\]

\[
C_{ij} = \frac{R_{ij} + C_{i+1,j+1}^{C}e^{2X_{ij}}}{1 + R_{ij}C_{i+1,j+1}^{C}e^{2X_{ij}}}
\]

with

\[
C_{N,N-1}^{N} = R_{N,N-1}^{N}
\]

\[
i = 2 \rightarrow N - 1
\]

\[
j = 1 \rightarrow i - 1 \quad (j = i - 1)
\]

\[
X_{i} = ik_{c} \cos \theta_{d}_{i}
\]

The \( R_{ij} \) are the Fresnel’s coefficients. They are given for both polarizations:

\[
R_{ij}^{P} = \frac{n_{i} \cos \theta_{j} - n_{j} \cos \theta_{i}}{n_{i} \cos \theta_{j} + n_{j} \cos \theta_{i}} \quad \text{P polarization}
\]

\[
R_{ij}^{S} = \frac{n_{j} \cos \theta_{j} - n_{i} \cos \theta_{i}}{n_{j} \cos \theta_{j} + n_{i} \cos \theta_{i}} \quad \text{S polarization}
\]
Using a prism as optical coupler, we can consider the mixed configuration of a layered system, as first described by Kretschmann [1] and Otto [2], and shown in Figure 1. For this configuration, the reflected signal for light p-polarization can exhibit two attenuations due to coupling to surface plasmons at both interfaces. To obtain the reflection dip, one can, for instance, vary the angle of the incident light, θ, beyond the critical angle.

Figure 1: Mixed configuration. The light couples to the sample layers through an evanescent field.

If both dielectrics are the same, the surface plasmons along either side of the metal film are coupled and their respective electric fields become large at interfaces for both angles for which the surface plasmons are excited. The electric field strength within the films can become large either in the metal or the dielectric layers. When the electric field becomes large inside the metallic layer, the wave will be dissipated quickly within the metal. Such a surface plasmon cannot propagate a long distance and for that reason has been called a Short Range Surface Plasmon – SRSP [3]. When the electric field becomes larger inside the dielectric layer it will suffer a smaller loss and the propagation distance of the surface plasmon will be longer, being therefore a Long Range Surface Plasmon - LRSP. For a given film system it becomes interesting to inquire whether the SP is coupled to the upper, the prism side, or lower, the airside, interface. With this goal in mind, we investigate where in the film occurs the coupling and the predominant power dissipation.

The electromagnetic field is calculated for both polarizations [4] and with this result the absorbed power density in each layer is determined according to:

\[ P_{\text{tot}} = \frac{1}{2} \text{Re} \left( \tilde{P} \cdot \tilde{E}^* \right) \]

where \( \tilde{P} = \epsilon_0 (1 - \epsilon) \tilde{E} \) represents the complex electric polarization.

We calculate both, the power absorbed and the radiant flux, S, for a specific coupling angle to a resonant mode and as a function of the depth within the layers.

The Poynting vector is given by

\[ S = \frac{1}{2} \text{Re} \left( \epsilon' \times \tilde{E} \right) \]

Results and Discussions

We calculate the electromagnetic fields for a 3-layer structure consisting of a BK-7 prism substrate (\( n_1=1.52 \)), a Na\(_3\)AlF\(_6\) film (\( n_2=1.33+0.003i \), \( d_2=350\)nm), a silver film(\( n_3=\sqrt{-16,17+0.9i} \), \( d_3=59\)nm) , another Na\(_3\)AlF\(_6\) film (\( n_4=1.33+0.003i \), \( d_4 \)) and finally air, considering a He-Ne laser beam (632.8 nm) incident from the prism side. The dielectric films are assumed having small losses, due to absorption or scattering. This way, it is possible to

Figure 2: Equi-reflectance curve of a symmetrical film dielectric / metal / dielectric, as a function of the incidence angle, varying the thickness of the last layer.
observe how the dissipation of the field in the dielectric layer behaves. The thickness of the last dielectric layer is varied in these calculations.

In Figure 2 we show the results for the calculated reflectance plotted as a contour map for this film sample, where the reflectance is function of the thickness of the outer dielectric film, d_4, and the incidence angle, θ, with p-polarized light. In this figure the clear regions correspond to high reflectivities, close to 100%, whereas the darker regions correspond to lower reflectivities, approaching even zero in the valleys, marked with dark black.

Initially, for a given thickness, d_4, we observe the presence of two minima in reflectivity, caused by the excitation of surface plasmons at the two interfaces of the metal. For a dielectric thickness greater than 300nm we can observe, in addition, the presence of another reflectivity minimum due to the first guided TM_1 mode. Notice that in the absence of the outer dielectric layer, by letting d_4 approach zero, we observe for lower incidence angle the SP being localized at the silver-air interface while for larger angle the SP occurs at the Ag-Na_3AlF_6 interface.

With increasing thickness d_4, the first reflectivity minimum moves to larger angles, while the effective refractive index increases. For a thickness of about 300nm the position of this situation inverts.

We can follow this behavior in the graph of the radiant flux, Sx, at the coupling angle, and as a function of the depth of the layers. In each one of the following graphs we also show the power absorbed in the dielectric layers (with losses) and in the metal. We consider the x-direction parallel to the interfaces and the z-direction perpendicular to them. Observing the x-component of the Poynting vector for the coupling angle, it is possible to

![Figure 3](image_url)

**Figure 3**: For small thickness of the last dielectric (300nm), the first minimum of reflection corresponds to plasmon excited in the second interface. Increasing the thickness (600nm), the minimum corresponds to plasmon excited in the first interface.

![Figure 4](image_url)

**Figure 4**: For small thickness of the dielectric (300nm), the last minimum of reflection corresponds to plasmon excited in the first interface. Increasing the thickness (600nm), the minimum corresponds to plasmon excited in the second interface.
identify which mode is resonant to the system. In the Figures 3 and 4, Sx is represented by a full line and Pot, the absorbed power density, as dotted line, both normalized.

In Figure 3 we show Sx and the power absorbed for the reflection minimum occurring at lower incidence angle, and, in Figure 4, for the larger angle, for two different thicknesses of the outer dielectric layer. It is observed that the energy flow is larger at the interface where the SP is excited.

We also notice that SP of each interface have interchanged their angular position, the lower angle corresponding now to the SRSP and the larger angle to the LRSBP. We also notice that the absorbed power follows the form of the energy flow. A high field entails large absorption while a low field has small absorption. With respect to the SP one observes that the absorption always is largest near the interface where plasmon is excited.

If a symmetrical film will be used as an optical sensor, which is expected to show high sensitivity to small refractive changes in the outer dielectric, the system must have the largest possible field in this layer. In this case the plasmon must be excited at the outer interface. A smaller film thickness must therefore to be chosen in order to monitor the reflection dip at the larger angle.

If such a film system would be used to produce an intensity dependent optical nonlinearity, the LRSP is indicated. This would allow for a larger path of interaction.

Conclusions
For our three-layer film system studied here we show that the spatial dependence on the power flow and/or power absorption across the layered film system reveals much more physical information than can be obtained by looking only at the overall reflectance, as measured in the ATR technique.

For optical sensor application films with the outer dielectric layer of small thickness, thinner than needed to excite the first waveguide mode, reveals itself more adequate. In our system, the reflection minimum at lower angle corresponds to a larger absorption is in the outermost layer, with higher angular dispersion, and is therefore more sensitive to slight modifications of this layer.

For non-linear optical interaction, where the highest field-intensity of is of importance, the choice will be the LRSP, corresponding to a SP excited at the larger angle for any arbitrary thickness.

Acknowledgements
The authors thank the FAPERGS and CNPq.

References