The Jaynes-Cummings model with a quantum external field

R.J. Missori †, *M.A. Marchiolli* *, *J.A. Roversi* †

† **Instituto de Física "Gleb Wataghin" - Universidade Estadual de Campinas**
13083-970 Campinas SP Brazil
***Instituto de Física de São Carlos - Universidade de São Paulo**
13560-970 São Carlos SP Brazil

Abstract

One of the simplest ways of studying the interaction of the radiation with the matter is through the Jaynes-Cummings model (JCM), that consists of an atom of two levels interacting with an electromagnetic field inside a cavity with high quality factor Q. This model has a great relevance in Quantum Optics, because it makes possible obtaining an exact analytical solution, within the dipole and rotate wave approximations (RWA). In the last years, several studies were realized on the system dynamics, where effects of collapses and revivals were observed in the inversion of atomic population, which proves the granular nature of the light and other associated nonclassical effects of the statistical properties of the field inside the cavity. With the help of the extensive literature on this topic, we verified that there is a model of nonlinear atomic homodyne detection, proposed by Wilkens and Meystre [1], that basically considers the JC molecule pumped by an intense coherent external field. In particular, the solubility of this model (cavity-atom-external field) becomes possible by means of the use of a semiclassical approximation, with the costs of severe restrictions on the time measurement of the physical observable associated to the external and internal fields. In this work, we overcame that limitation and solved the model analytically for any fields in consideration (principally the quantized external field). Specifically, we will show how the external field can be used to sustain the quantum properties of the field in the cavity, besides other important effects.

In April of 1991, Martin Wilkens and Pierre Meystre [1] proposed an atomic homodyne detection technique with the objective of detecting multiphoton coherences coming from quantum macroscopic superpositions generated in superconductors micromaser cavities (this scheme is a nonlinear version of homodyne detector of a single atom). Basically, the model consists of a quantum electromagnetic field (mode "a") confined in a cavity of high quality factor Q and one intense electromagnetic field (mode "b"), both interacting with a two level atom. The hamiltonian operator of this system considers the atom resonant with both fields and equals coupling constants, in other words,

$$\boldsymbol{H} = \hbar\omega\, \boldsymbol{a}^\dagger \boldsymbol{a} + \hbar\omega\, \frac{\boldsymbol{\sigma}_z}{2} + \hbar\omega\, \boldsymbol{b}^\dagger \boldsymbol{b} + \hbar\kappa(\boldsymbol{a}\boldsymbol{\sigma}_+ + \boldsymbol{a}^\dagger\boldsymbol{\sigma}_-) + \hbar\kappa(\boldsymbol{b}\boldsymbol{\sigma}_+ + \boldsymbol{b}^\dagger\boldsymbol{\sigma}_-), \tag{1}$$

where $\hbar$ is the Planck constant, ω is the frequency of the fields external, in the cavity and of the atom, and κ is the coupling constant of the atom with both fields. Because of the simplification of the model, the authors assume that the fields oscillation frequencies are the same, as well as the coupling constants of the atom with both fields. Here, the atomic operators $\boldsymbol{\sigma}_\pm$ and $\boldsymbol{\sigma}_z$ refers the Pauli matrices for the two level atom and, last, $\boldsymbol{a}^\dagger(\boldsymbol{a})$ and $\boldsymbol{b}^\dagger(\boldsymbol{b})$ are the operators creation (annihilation) of photons of the fields in the cavity and external, respectively. This way, considering the atom in the excited state, the field in the cavity in the coherent state and the external field as being a field coherent intense (besides other mathematical artifices that can be seen in [1]), we have that the ionization probability, which can be used to reconstruct the Wigner function of the field in the cavity, is given by

$$p(\tau) = \frac{1}{2} + \frac{1}{2}\exp\left[-\frac{1}{2}(\kappa\tau)^2\right]\cos\left(2\kappa\tau|\alpha+\beta|\right). \tag{2}$$

However, an important restriction exists in what refers the validity of the approach used in the calculation of the expression above. For this to be valid it is necessary to assure that the time of interaction τ between the atom and the fields are enough short in way to allow to despise the effects of vacuum fluctuations. This implicates that we are working with a time τ much smaller than $1/\kappa$ and, consequently, assuring that the spontaneous transitions of Rabi (vacuum fluctuations) will not happen. Therefore, the scheme of nonlinear atomic homodyne detection proposed by Wilkens and Meystre just supplies partial information about the field contained in the cavity, having in view that the values of the characteristic Wigner function, that is connected with the ionization probability, are restricted to $\kappa\tau \ll 1$. However, Dutra et al. [2] got to obtain a quantum expression without considering the approach used in [1] for the ionization probability, in other words,

$$p(\tau) = \frac{1}{2} + \frac{1}{2} e^{-|\alpha+\beta|^2/2} \sum_{n=0}^{\infty} \left[\frac{|\alpha+\beta|^2}{2} \right]^n \frac{1}{n!} \cos\left(2\sqrt{2}\kappa\tau\sqrt{n+1}\right), \tag{3}$$

that, for comparison with the results of Wilkens and Meystre, in the limit of $|\beta| \gg 1$, that takes us to the asymptotic approach

$$p(\tau) = \frac{1}{2} + \frac{1}{4} \exp\left(-\frac{1}{2}|\alpha+\beta|^2\right) \left[e^{i\kappa\tau|\alpha+\beta|} \exp\left(\frac{1}{2}|\alpha+\beta|^2 e^{2i\kappa\tau/|\alpha+\beta|}\right) + c.c \right]. \tag{4}$$

However, the methodology used to obtain that expression turns obligatory the fields of the cavity and external to be coherent, besides equal coupling constants. Moreover, the same authors [3] proved that the quantum fluctuations of the external field, upon important time scales, can be neglectful since the atom is faintly coupled to the external field and strongly coupled with the field of the cavity. Thus, it can be obtained a more precise measurement of the field state through the detection of the atoms that leave the cavity in the excited state. It is important we notice that now have two coupling constants: κ_a for the field in the cavity and κ_b for the external field. In this work, we obtained the exact analytic solution for the hamiltonian operator

$$\boldsymbol{H} = \hbar\omega\, \boldsymbol{a}^\dagger \boldsymbol{a} + \hbar\omega_0 \frac{\boldsymbol{\sigma}_z}{2} + \hbar\omega\, \boldsymbol{b}^\dagger \boldsymbol{b} + \hbar\kappa_a(\boldsymbol{a}\boldsymbol{\sigma}_+ + \boldsymbol{a}^\dagger \boldsymbol{\sigma}_-) + \hbar\kappa_b(\boldsymbol{b}\boldsymbol{\sigma}_+ + \boldsymbol{b}^\dagger \boldsymbol{\sigma}_-) \tag{5}$$

for any values of the coupling constants and with the frequencies of the fields not necessarily the same as the frequency of the atom. This way, the ionization probability obtained for this case is

$$p(\tau) = \frac{1}{2} + \frac{e^{-|\gamma_{\text{eff}}|^2}}{2} \sum_{n=0}^{\infty} \frac{|\gamma_{\text{eff}}|^2}{n!} F_n(\tau), \tag{6}$$

where

$$\kappa_{\text{eff}}^2 = \kappa_a^2 + \kappa_b^2\,, \qquad \omega_n = 2\kappa_{\text{eff}}\sqrt{n+1}\,, \qquad \Delta_n^2 = \delta^2 + \omega_n^2\,, \qquad \gamma_{\text{eff}} = \frac{\kappa_a\alpha + \kappa_b\beta}{\kappa_{\text{eff}}},$$

$$F_n(\tau) = 1 - 2\frac{\omega_n^2}{\Delta_n^2} \sin^2\left(\frac{\tau\Delta_n}{2}\right).$$

In the Figure 1 we can observe the difference for each one of the cases described in this work. For all the graphs we considered the atom initially in the excited state and both fields in the coherent state. We observed that the fact of we could use a quantum external field allows us to observe nonclassical phenomena for times smaller, that is, times more realists from the experimental point of view. This was possible without we consider any restriction as for the coupling constants (that in this case were considered the same), as for the frequency of the atom and of the fields, or as for the amplitude of the external field. Several other initial configurations can be considered as, for instance, different fields in the cavity. For the case of the field in the thermal cavity, interesting results exist on the literature (for the case of the classical approach)[4]. However, only for the configurations used in this work we observed significant results and which can be better explored.

In future work, we intended to explore in full detail the influence of the detuning and of different coupling constants in the dynamics of the system for both coherent fields, as well as for other fields in the cavity. With this, we will can study the decoherence effects on quantities that characterize the field inside the cavity, besides other relevant alterations that an external quantum field can cause in the system.

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References

[1] M. Wilkens and P. Meystre, *Nonlinear atomic homodyne detection: A technique to detect macroscopic superpositions in a micromaser*, Phys. Rev. A **43**, 3832 (1991).

[2] S. M. Dutra, P. L. Knight and Moya-Cessa, *Discriminating field mixtures from macroscopic superpositions*, Phys. Rev. A **48**, 3168 (1993).

[3] S. M. Dutra and P. L. Knight, *Atomic probe for quantum states of the electromagnetic fields*, Phys. Rev. A **49**, 1506 (1994).

[4] Fu-li Li and Shao-yan Gao, *Controlling nonclassical properties of the Jaynes-Cummings model by an external coherent field*, Phys. Rev. A **62**, 043809 (2000).

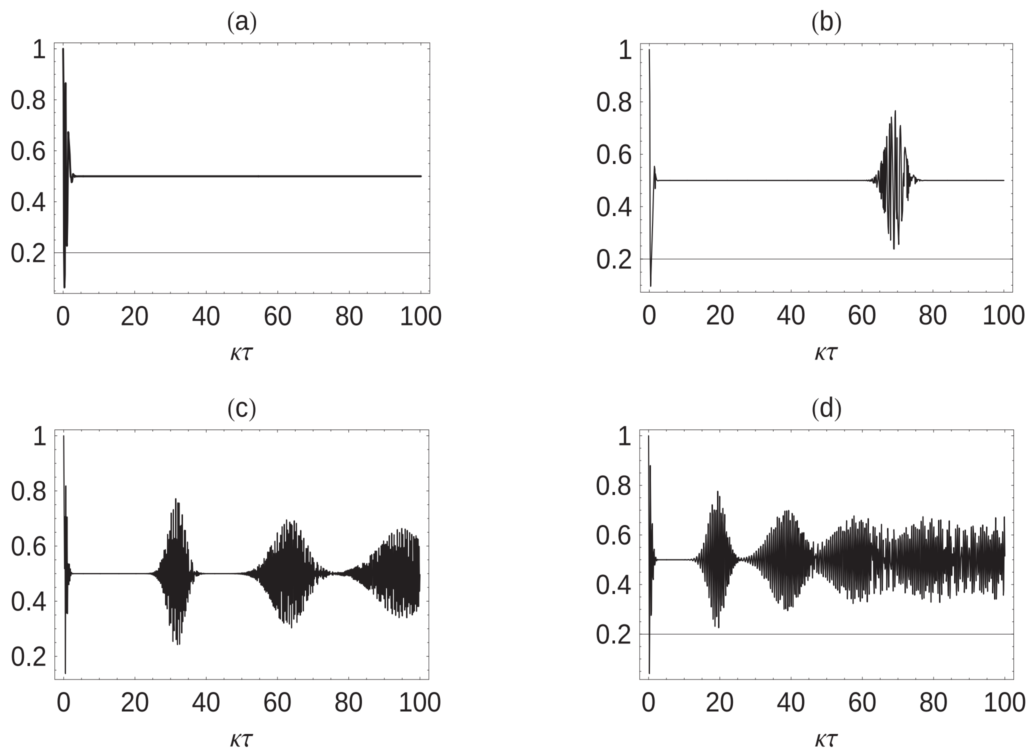

Figure 1: Graphs of the ionization probability . (a) The graph refers the Eq. (2), where the approach of the classic external field was considered. (b) That graph is regarding Eq. (4), in other words, the exact calculation of the excitation probability. They were considered here $\alpha = 2$ and $\beta = 20$ (intense external field). We can observe a revival around $\kappa\tau = 65$. Comparing that graph with the previous, we observed that the approach accomplished in the graph (a) omits important results, as we already commented in the text. It is important to notice that this graph can also be obtained through Eq. (6), if we consider $\delta = 0$. (c) Here the graph is regarding Eq. (6), in that any approach type doesn't exist. We considered $\alpha = 2$ and $\beta = 8$. We can notice that the time of appearance of the collapses and revivals decreased enough if compared to the previous graphs. (d) This graph also refers the Eq. (6), with $\alpha = 2$ and $\beta = 4$. If we compare all the cases, we verified that the fact of considering the external field not very intense it turns possible we observe nonclassical phenomena in a shorter time. It is important to point out that for values of β very small (smaller than 4), we began to lose the definition of the collapses and revivals.