Propagation of the Angular Spectrum in the Up-conversion

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Abstract

We study the propagation of the angular spectrum of the light produced in the up-conversion process. The connection between the angular spectrum of the fundamental and the second harmonic is observed. We show that even though they are connected, in general it is not possible to directly transfer images from the fundamental to the second harmonic. We also analyze the up-conversion of a Laguerre-Gaussian mode in a long crystal regime. In this case, because of the phase matching conditions and the walk-off, the outgoing second harmonic beam is not in a Laguerre-Gaussian mode anymore. We investigate the possibility of a residual orbital angular momentum in the second harmonic beam

Introduction

In the parametric up and down-conversion processes[1], the converted fields are connected to the pumping fields through the phase matching conditions. It has been shown, for example, that the entangled field of the twin photons from the down-conversion carries the same angular spectrum as the pump[2]. It was also shown that in the stimulated down-conversion, the idler beam may carry the same angular spectrum as the pump or the complex conjugated of the stimulating beam[3]. This transfer of the angular spectrum implies in the transfer of images formed by those beams and also in the transfer of the orbital angular momentum[4,5,6].

In this work, we study the propagation of the light fields and their coupling in the SHG process, taking into account their transverse spatial distributions, or in other words, propagating their angular spectra. We demonstrate how the angular spectrum of the fundamental and the second harmonic fields are connected. It is shown that even though they are connected, this connection does not allow the direct transfer of images from one field to the other. It has been shown by Firester[7], that in the special case where one image is formed by the fundamental inside the crystal, the second harmonic will reproduce this image at the same plane, even though they do not have exactly the same angular spectrum. On the other hand, as the transfer function is known, it is possible to manipulate the second harmonic angular spectrum, by preparing the fundamental.

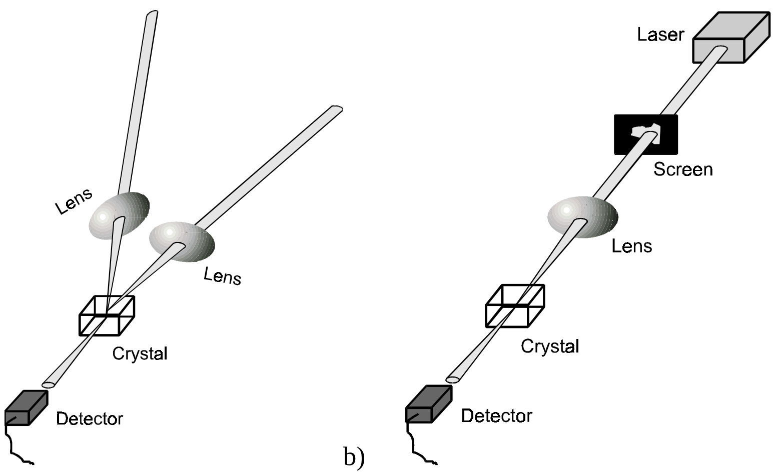

Fig. 1 - Sketch of the experiment. a) Non-collinear up-conversion. b) Second harmonic generation.

Let us consider the experimental set-up sketched in Fig.1. In the non-collinear case (Fig.1a) two different modes pump a non-linear crystal for generating a third mode, the up-converted mode. SHG is the collinear version (Fig.1b), where just one mode pumps the nonlinear crystal and the degenerate collinear up-conversion occurs. Using the paraxial approximation, it is possible to demonstrate that the intensity in a plane transverse to the second harmonic propagation is given by:

$$I(\rho) \propto \left| \int d\rho' \, W_1(\rho') \, W_2(\rho') \, \exp\left[i \left| \rho - \rho' \right|^2 \frac{k}{2z} \right] \right|^2 \quad (1)$$

where $W_1(\rho')$ and $W_2(\rho')$ are the field transverse distributions for two non-collinear fundamental pumping beams, k is the wave-number for the second harmonic and z is the distance from the crystal to the observation plane.

For the special case where modes 1 and 2 are the same (collinear case), the above equation is simplified to

$$I(\rho) \propto \left| \int d\rho' \, W^2(\rho') \; \exp\left[i \left| \rho - \rho' \right|^2 \frac{k}{2z} \right] \right|^2 \quad (2)$$

From the equations above it is seen that the transverse field of the second harmonic is given by the square of the fundamental, propagated from the crystal to the observation plane. It is possible to calculate the intensity profile of the second harmonic for a given fundamental input. We are not going to present these calculations here, but they will be plot together with the experimental results.

We have performed measurements of the transverse intensity distribution for the fundamental and the second harmonic in the SHG, in different propagation planes after the crystal.

Experimental Setup

An infrared diode laser, with wavelength centered around 850 nm, 150 mW output power is directed to a $LiIO_3$ (Lithium Iodate) non-linear crystal. SHG takes place, producing a beam with wavelength around 425 nm. Before the infrared reaches the crystal, it is passed through a double-slit diffraction screen with slits width a = 0.2mm, separation d = 0.4mm and also through a thin lens with f = 10 cm focal length. After the crystal, fundamental and second harmonic beams are separated by a prism and they are both detected with single photon counting modules, based on cooled avalanche photodiodes. The photon counting is necessary for the second harmonic beam, because the signal level is strongly decreased by the passage of the fundamental through the double-slit, which also decreases the efficiency of the up-conversion. The pulses coming from the detectors are sent to photon counters controlled by a computer which performs the data acquisition.

Results and Discussions

In a first set of measurements we have placed the slits at 12.3cm before the lens, so that the image of the slits was formed in a plane situated at 53.4cm after the lens. The crystal is always in the focal plane of the lens, so that the detection plane is 43.4cm from its center. The detectors are scanned in the vertical direction and the intensity profile is registered for both the fundamental and the second harmonic signals. Fig.2 shows these profiles. The fundamental presents an extended image of the slits, as it was expected. According to our calculations the intensity pattern for the second harmonic corresponds to the self convolution of the fundamental' s. See Fig.3. We believe that measurement and calculation are in good agreement, even though the measurement procedure with a finite detection slit smooths the experimental curve. This result shows that images may not be, in general, directly transferred from the fundamental to the second harmonic. On the other hand, the basic information about the image is contained in the second harmonic pattern and it might be recovered by de-convolution.

In a second set of measurements, the slits were placed 2cm before the crystal so that no real image was formed for the fundamental. The results are shown in Fig.4a for the fundamental and Fig.4b for the second harmonic. Once again the patterns are different. The calculation for the second harmonic intensity distribution is shown in Fig.5.

In a third set of measurements, we have removed the slits and prepared the fundamental in a l=1, p=0 Laguerre Gaussian mode. It is known that for a thin crystal or non-critical phase matching, the second harmonic results in a Laguerre-Gaussian mode with l=2, p=0 [6]. In our experiment, however, we have used a thick crystal in critical phase matching conditions. The second harmonic intensity distribution obtained is shown in Fig.6a. We have tried to determine weather the second harmonic beam had some orbital angular momentum, or not. In order to see this, we have sent it through a Michelson interferometer, in the same fashion as in Ref. [5]. The result is shown in Fig.6b. These results indicate that the orbital angular momentum is lost. Calculations for this kind of interaction are rather complicated and have not yet been performed. Qualitatively, the Laguerre-Gaussian mode is destroyed by the walk-off, that is why the resulting mode presents stripes in the walk-off direction, instead of round doughnuts.

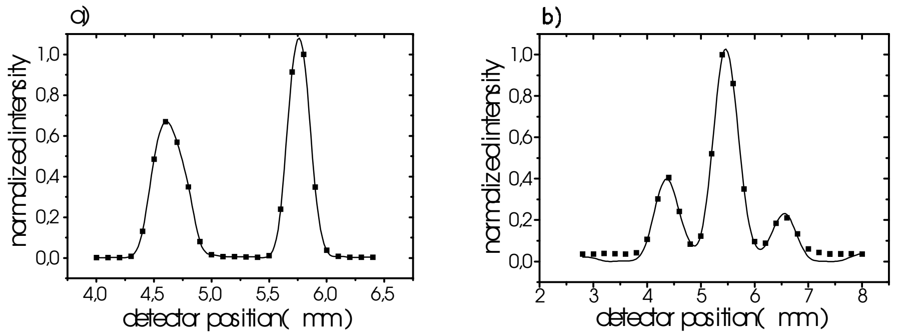

Fig.2 - Experimental intensity patterns. a) Fundamental, image plane. b) Second harmonic, image plane.

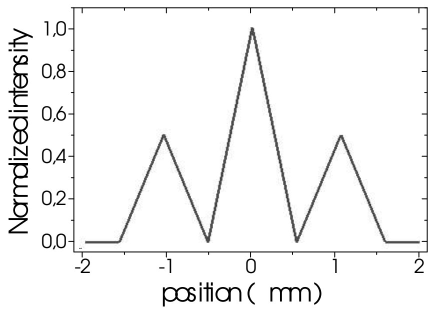

Fig.3 - Self-convolution for a two-slits distribution.

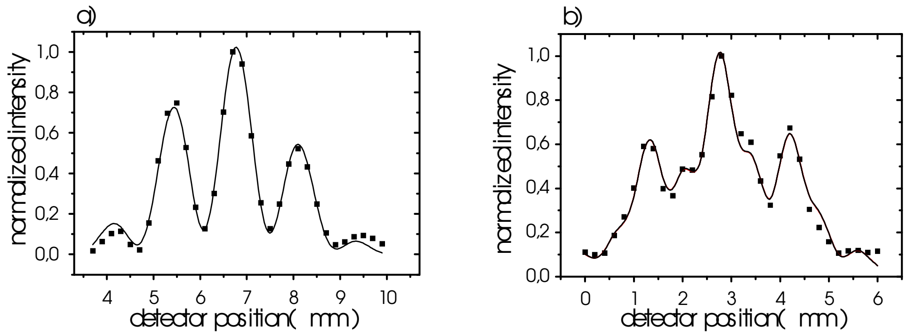

Fig. 4 - Experimental intensity patterns. a) Fundamental. b) Second harmonic.

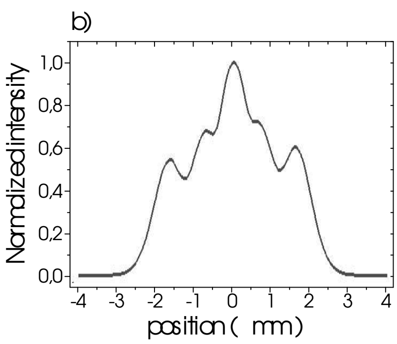

Fig. 5 - Theoretical intensity pattern for the second harmonic.

a)

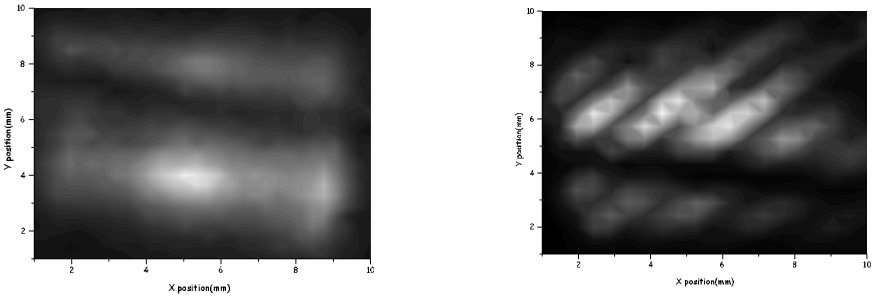

Fig. 6 - Experimental intensity patterns. a) Second harmonic. b) Interference in a Michelson interferometer.

Conclusions

In conclusion, we have demonstrated how the angular spectrum of the light produced in the second harmonic generation process is related to the fundamental one. Images cannot be directly transferred from the fundamental to the second harmonic, except in one special case. We also show that for a thick crystal, the up-conversion of a Laguerre-Gaussian beam results in a different mode, because of the walk-off. Experimental results indicate that the orbital angular momentum is not transferred from the fundamental to the second harmonic.

Acknowledgements
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