Quantum state engineering: comparing different schemes creating states of light field in traveling waves.

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Abstract

In this contribution we investigate the interaction of a single ion in a trap with laser beams. Our approach, based on unitary transformating the Hamiltonian, allows its exact diagonalization without performing the Lamb-Dicke approximation. We obtain a transformed Jaynes-Cummings type Hamiltonian, and we demonstrate the existence of super-revivals in that system.

1 Introduction

Generation of states of the quantized light field (or atomic oscillators), named "quantum states engineering" (QSE), turned out to be a very important topic of Quantum Optics [1] and Atomic Physics [2] in recent years. The issue has interesting potential applications, as in teletransport[3], quantum computers[4], quantum cryptography [5], quantum lithography [6], etc. Besides these practical applications it is also useful in fundamentals of quantum mechanics, as generation of "entangled states" [7] and Schrodinger's cat states [8]; decoherence of mesoscopic superpositions [9]; etc.

It is worth mentioning that, even apparent "exotic" states may become crucial in the determination of certain properties of a given system. To give some exemples, we cite the "reciprocal binomial state" (RBS), decisive for the experimental determination of the phase-distribution $P(\theta)$ of an arbitrary state [10] and quantum lithography [6]. The same role is played is by the "polynomial state", crucial for the experimental determination of the Husimi Q-function [11].

2 Distinct Schemes of QSE

2.1 Quantum-Scissors Scheme

A recent proposal was presented by Pegg, Phillips and Barnett [12] for preparation of an arbitrary running-wave superposition of the vacuum and one-photon states, $|\Psi_1\rangle = C_0 |0\rangle + C_1 |1\rangle$. In this scheme, a traveling field would be available for further applications, as auxiliary to determine properties of other field states discribing a system. The scheme is able to achieve the above mentioned superposition by a physical truncation of the photon number superposition making up a coherent state. The proposal requires no additional extension of current experiments and is reasonably insensitive to photodetection efficiency for the fields most likely to be used in practice. Since the scheme produces a truncation of the Hilbert space, it has been called "quantum scissors" device. As mentioned in [12], states with higher photon number might be constructed by superposing fields prepared as superpositions of zero and one photon number states.

More recently, Villas-Boas et al. [13] have extended the scheme of Ref.[12] to the case: $|\Psi_N\rangle_a = \sum_{n=0}^{N} C_n |n\rangle$. Here we present a brief summary of the procedure of Ref.[13], for the particular case $C_n = 1$. To recover the situation considered in [12] we assume that the quantum state to be generated is a finite superposition of equally weighted Fock states: $|\Psi_N\rangle_a = |0\rangle + |1\rangle + |2\rangle + ... + |N\rangle$. We will assume the scheme sketched in Fig.1, with the input state entering the splitter BS1 given by $|\Psi_{in}\rangle_{ab} = |1\rangle_a |N-1\rangle_b$. In this way, we have the output of BS1, $|\Psi_{out}\rangle_{ab} = \hat{R}_{ab} |\Psi_{in}\rangle_{ab}$, where $\hat{R}_{ab}$ is the unitary operator

$$\hat{R}_{ab} = \exp\left[i\theta_1(\hat{a}^\dagger \hat{b} + \hat{a}\hat{b}^\dagger)\right] \tag{1}$$

and $\theta_j = \tan^{-1}(r_j/\ t_j)$, with $t_j = \cos(\theta_j)$, $r_j = \sin(\theta_j)$ standing respectively for the reflection and transmission coefficients of beam splitters $BSj,\ j = 1, 2$. Next, the input of BS2, $|\Psi_{in}\rangle_{bc} = |\Psi_{out}\rangle_{ab} |\Psi_{in}\rangle_c$, with $|\Psi_{in}\rangle_c = \sum_{n=0}^{\infty} \gamma_n |n\rangle$; consequently, the output state emerging from the BS2 reads $|\Psi_{out}\rangle_{abc} = \hat{R}_{bc}(|\Psi_{out}\rangle_{ab} |\Psi_{in}\rangle_c)$. By measuring the field mode $\underline{\mathbf{b}}$ in the state $|1\rangle_b$ and the field mode $\underline{\mathbf{c}}$ in the state $|N-1\rangle_c$, we synthesize the projection of the field mode $\underline{\mathbf{a}}$ in the desired superposition. Once we have specified the state to be prepared, this implies a system of N equations. The solution of such a system can be guaranteed if the number of equations is not greater than the number of free variables. Therefore, we stress that for the present generalized quantum scissors [13], θ_1 and θ_2 are free parameters to be adjusted for the achievement of the desired state. When $N > 3$ we must introduce $N - 3$ new parameters to permit solubility of a system of coupled equations. This goal is attained by substituting the auxiliary coherent state field $|\Psi_{in}\rangle_c$, entering the BS2 by a (convenient) discrete superposition of coherent states, as also implemented in [13]. So, we may write

$$\gamma_n = \exp\left(-\frac{|\alpha|^2}{2}\right) \frac{|\alpha|^n}{\sqrt{n!}} \beta_N \tag{2}$$

where,

$$\beta_N = \begin{cases} 1,\ if\ \ N\ \ \leq 3 \\ \sum_{m=4}^{N} \cos(n\phi_m),\ \ if\ \ \ N > 3 \end{cases} \tag{3}$$

and the ϕ_m are the additional parameters to be determined for generation of the desired state.

In this scheme the probability P to produce a state $|\Psi_N\rangle_a$ is determined by the requeriment that the detectors D_b and D_c register, $N - 1$ and 1 photons, respectively. So, it is given by

$$P = \left|{}_c\langle 1|_b \langle N-1| \, |\Psi_{out}\rangle_{abc}\right|^2, \tag{4}$$

where,

$$\begin{aligned} {}_c\langle 1|_b \langle N-1| \, |\Psi_{out}\rangle_{abc} &= \frac{1}{\sqrt{(N+1)!}} \sum_{q=0}^{N-1} \sum_{r=0}^{q} \sum_{n=0}^{N} \sum_{p=0}^{n} \begin{pmatrix} N-1 \\ q \end{pmatrix} \begin{pmatrix} q \\ r \end{pmatrix} \begin{pmatrix} n \\ p \end{pmatrix} \\ &\times i^{(N+p-r+1)}\, t_1^q\, t_2^{(r+n-p)}\, r_1^{(N-1-q)}\, r_2^{(q-r+p)}\, \frac{\gamma_n}{\sqrt{n!}}\, (A+B+C) \end{aligned} \tag{5}$$

with

$$\begin{aligned} A &= t_1 \sqrt{(r+p)!} \sqrt{(q+n-r-p)!} \sqrt{(N-q)!} \delta_{(1\,,\,r+p)} \delta_{(N-1\,,\,q+n-r-p)} |N-q\rangle_a \\ B &= -r_1 r_2 \sqrt{(r+p)!} \sqrt{(q+n-r-p+1)!} \sqrt{(N-1-q)!} \delta_{(1\,,\,r+p)} \delta_{(N-1\,,\,q+n-r-p+1)} |N-1-q\rangle_a \\ C &= i r_1 t_2 \sqrt{(r+p+1)!} \sqrt{(q+n-r-p)!} \sqrt{(N-1-q)!} \delta_{(1\,,\,r+p+1)} \delta_{(N-1\,,\,q+n-r-p)} |N-1-q\rangle_a . \end{aligned} \tag{6}$$

and r_i, t_i being the reflection and transmission of beam splitters.

2.2 Dakna's Scheme

Another representative scheme of **QSE** to generate states of a light field in a running-wave is one by Dakna et al [14]. In this scheme the state (following the notation of [14])

$$|\Psi\rangle \simeq \sum_{n=0}^{N} \Psi_n |n\rangle, \tag{7}$$

with given Ψ_n, is written in the form

$$|\Psi\rangle \simeq \left(\sum_{n=0}^{N} \frac{\Psi_n (\hat{a}^\dagger)^n}{\sqrt{n!}} \right) |0\rangle \tag{8}$$

which, for convenience, can also be rewritten as

$$|\Psi\rangle = \frac{\Psi_N}{\sqrt{N!}} (\hat{a}^\dagger - \beta_N^*)(\hat{a}^\dagger - \beta_{N-1}^*).....(\hat{a}^\dagger - \beta_1^*) |0\rangle \tag{9}$$

where the β_i are the roots of the polynomial equation

$$\sum_{n=0}^{N} \Psi_n \beta^n = 0. \tag{10}$$

Next, the Eq.(7) is interesting for manipulation in the set of beam-splitter of Fig. 2. Using the well known relation

$$(\hat{a}^\dagger - \beta^*) = \hat{D}(\beta)\hat{a}^\dagger \hat{D}^\dagger(\beta) \tag{11}$$

where $\hat{D}(\beta)$ stands for the displacement operator, the substitution of (11) in (9) gives

$$|\Psi\rangle = \left\{ \left[\hat{D}(\beta_N)\hat{a}^\dagger \hat{D}^\dagger(\beta_N) \right] \left[\hat{D}(\beta_{N-1})\hat{a}^\dagger \hat{D}^\dagger(\beta_{N-1}) \right] ... \left[\hat{D}(\beta_1)\hat{a}^\dagger \hat{D}^\dagger(\beta_1) \right] \right\} \ |0\rangle . \tag{12}$$

Now, assuming zero detection in the detectors D_1, D_2, ..., D_N and $|\Psi_1\rangle = D(\alpha_1)\ |0\rangle$, we obtain, step-by-step: $|\Psi_2\rangle = \hat{a}^\dagger T^{\hat{n}} |\Psi_1\rangle = \hat{a}^\dagger T^{\hat{n}} D(\alpha_1)\ |0\rangle$, $|\Psi_3\rangle = D(\alpha_2)(\hat{a}^\dagger T^{\hat{n}} D(\alpha_1)\ |0\rangle)$ ... and

$$|\Psi_N\rangle = \{\ [D(\alpha_N)(\hat{a}^\dagger T^{\hat{n}} D(\alpha_{N-1})]...[D(\alpha_1)]\ \}\ |0\rangle . \tag{13}$$

Of course, the Eq.(13) differs from the Eq.(12). However, they can be connected using sucessive action of each pair of neighboring beam-splitters, as follows

$$[D^\dagger(\alpha)T^{\hat{n}} D(\alpha)]\hat{a}^\dagger = T\hat{D}^\dagger(\bar{T}^*\alpha)\hat{a}^\dagger \hat{D}(\bar{T}^*\alpha)[D^\dagger(\alpha)T^{\hat{n}} D(\alpha)] \tag{14}$$

where $\bar{T} = 1 - T^{-1}$. Next, after using the Eq.(14) in the Eq.(13) and comparing this results with Eq.(12) shows that they become identical when $\alpha_1 = -\sum_{l=1}^{N} T^{-l}\alpha_{l+1}$ and

$$\alpha_k = T^{*N-k+1}(\beta_{k-1} - \beta_k) \tag{15}$$

for $k = 2, 3, 4...N$, yielding the experimental parameter α_k.

The probability $P(\Psi)$ to produce our desired state $|\Psi\rangle$ is given by [14]

$$P(\Psi) = \left\| \hat{Y} D(\alpha_N)\hat{Y} D(\alpha_{N-1}) .... \hat{Y} D(\alpha_1)\, |0\rangle \right\|^2 , \tag{16}$$

where $\hat{Y} = R\hat{a}^\dagger T^{\hat{n}}$. That is, probability is given by the square of the norm of the state produced when no photon is registered in each of the N conditional output measurements. After some algebra on the Eq.(16) we obtain

$$P(\Psi) = |R|^{2N} |T|^{N(N-1)} \left\| \Pi_{m=1}^{N} (\hat{a}^\dagger + b_{mN}^*)\, |\gamma_N\rangle \right\|^2 \exp\left(-|R|^2 \sum_{m=1}^{N} \left| \sum_{j=1}^{m} T^{m-1}\alpha_j \right|^2 \right) \tag{17}$$

where $b_{1N} = 0$, $b_{mN} = -\sum_{j=0}^{m-2} T^{*-j-1}\alpha_{N-j}$, $m = 2, 3, ...N$, $\gamma_N = \sum_{j=1}^{k} T^{k+1-j}\alpha_j$ and

$$\left\| \Pi_{n=1}^{N} (\hat{a}^\dagger + b_{mN}^*)\ |\gamma_N\rangle \right\|^2 = \sum_{m,l=0}^{N} B_{N,m}(0)\, B_{N,l}^*(0)\ \langle\gamma_N|\ \hat{a}^{N-m}\, \hat{a}^{\dagger\,(N-l)}\ |\gamma_N\rangle \tag{18}$$

with,

$$B_{N,\,p}(0) = \left[\frac{1}{(N-p)!} \frac{d^{N-p}}{dx^{N-p}} \left(\prod_{i=1}^{N} (x + b_{iN}) \right) \right]_{x=0} . \tag{19}$$

3 Comparison of efficiences

In the scheme of Ref.[12] the probability P for $N = 1$ is obtained as $P_1 = \frac{1}{4}\sum_{n=0}^{1} |C_n|^2$, which results $P_1 = 1/2e \simeq 18\%$, for $\alpha = 1$ and assuming an equally weighted superposition ($C_0 = C_1 = 1/\sqrt{2}$). In the extended scheme of [13] we obtain for $\alpha = 1$ and equally weighted superposition: $P_2 \cong 9\ \%$, for $N = 2$; $P_3 \cong 9\ \%$ for $N = 3$; $P_4 \cong 3,7\ \%$ for $N = 4$. However, these probabilities are only apparent. One should correct them by taking into account the probability of having the required input state, namely: for the "quantum scissors" we might also consider the probability to have the required input state $|N-1\rangle$ in the mode $\underline{\mathbf{b}}$ and the input state $|\alpha\rangle$ (or a superposition $\sum_i |\alpha_i\rangle$) in the mode $\underline{\mathbf{c}}$. The state $|N-1\rangle$ is available with probability $P \cong 20\%$ [15], whereas the state $|\alpha\rangle$ is available with maximum efficience $P = 100\%$ [16]. For $N > 3$ superposition of coherent states are required in the mode $\underline{\mathbf{c}}$ [13]. In this case, the probability depends on the phase ϕ between the coherent components [16]. For equally weighted superpositions as employed in [13], $\phi \simeq 8.8$; hence we obtain, using [16], $P \simeq 50\%$. Next, multiplying the previous probabilities by the foregoing corrections we obtain the resulting probabilities, as shown in the first row of the table-1.

Next, consider the scheme proposed by Dakna et al [14], as depicited in Fig.2. The detectors are also assumed ideals. Note that for $N = 1$ the difference between the arrangements of [12] and [14] relies on the detection methods. In [14], additional beam splitters are required when $N \geq 2$. Although no applications for equally weighted superpositions were considered in [14], a straightforward application of its scheme furnishes the efficiences for generation of the equally weighted superpositions: $|\Psi_N\rangle = \sum_0^N C_n |n\rangle$, for $N = 2$, $N = 3$ and $N = 4$. In this case we find, for $\alpha = 1$: $P'_2 \cong 14$ % for $N = 2$; $P'_3 \cong 5$ % for $N = 3$; $P'_4 \cong 1.5$ % for $N = 4$.

N	1	2	3	4
P_N (%)	18	1,8	0,9	0,6
P'_N (%)	40	14	5	1,5

Table-1. Probabilities for the Quantum Scissors scheme (P_N) and for the Dakna's scheme (P'_N) for $N = 1, 2, 3, 4$.

4 Generation of the Reciprocal Binomial State

The relevance a traveling mode prepared in the RBS in traveling fields was mentioned in the Sect. I. In this section we investigate its generation using the Dakna's scheme. The RBS is defined as

$$|RBS\rangle = C \sum_{k=0}^{N} \begin{pmatrix} N \\ k \end{pmatrix}^{1/2} \exp\left[ik\left(\phi - \frac{\pi}{2}\right)\right] |k\rangle , \qquad (20)$$

where $\begin{pmatrix} N \\ k \end{pmatrix}$ stands for the binomial coefficient and C is a normalization constant. To be specific, we set $N = 5$, $\phi = \pi$. By indentifying the coefficients Ψ_n in Eq.(7) with $C \sum_{k=0}^{N} \begin{pmatrix} N \\ k \end{pmatrix}^{1/2} \exp\left[ik\left(\phi - \frac{\pi}{2}\right)\right]$ of Eq.(20), we obtain the values of parameters $|\beta_k|$, φ_k, $|\alpha_k|$ and θ_k appearing in the Dakna's scheme yielding the RBS. In this case obtain maximun probability $P = 0,2\%$ when $T = 0,87$ (cf. Fig.3)

k	$\lvert\beta_k\rvert$	φ_k	$\lvert\alpha_k\rvert$	θ_k
1	1.00	-3.11	0.03	1.25
2	1.00	-1.93	0.64	2.19
3	1.00	1.54	1.30	-1.76
4	1.00	-0.78	1.39	1.95
5	1.00	0.36	0.94	-1.78
			1.00	0.36

Table-1. Values of parameters $|\beta_k|$, φ_k, $|\alpha_k|$, θ_k to generate the RBS, with $\beta_k = |\beta_k| \exp(i\varphi_k)$, $\alpha_k = |\alpha_k| \exp(i\theta_k)$.

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