Image-polarization entanglement in parametric down-conversion

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Abstract

We demonstrate both theoretically and experimentally the entanglement between quantum images and polarization in twin photons generated by parametric down conversion with two crystals. This configuration has been proved to be an efficient source of Einstein-Podolsky-Rosen (EPR) polarization states. We now add the transverse degrees of freedom as a further source of entanglement. Potencial applications to quantum cryptography of images will be discussed.

The increasing interest on quantum information processing and communication has brought the need for efficient sources of etangled states of a quantum system. In the optical domain, the production of polarization entangled photon pairs in type II parametric down-conversion [1] has allowed many progresses in this matter. More recently, a two-crystal geometry has been employed yielding a bright source of polarization-antangled photons [2]. This source was composed by two adjacent down-conversion (BBO) crystals cut for type-I phase matching. The photon-pairs created by pumping these crystals are well described by a polarization-entangled quantum state of the kind $HH + e^{i\phi}VV$. The whole family of "Bell states" can be obtained by introducing half-wave plates on signal and idler beams. As described in ref. [2], the polarization entanglement is measured through polarization analysis of the down-converted photon pair. The potential applications of such a source of entangled photons are inumerous and a different aspect of this setup was exploited in ref. [3]. There, polarization entanglement was measured through position interference. In this work, we present a multimode theory which suggests a scheme for entangling images to the polarization degree of freedom.

The multimode quantum state generated by the two-crystal configuration can be derived taking into account the angular spectrum of the pump beam. Both crystals will be considered as a box with dimensions L_x, L_y and L_z. The crystals are labeled 1 and 2. Crystal 1 is centered at the origin of the coordinate system while crystal 2 is centered at $\mathbf{d} = d\hat{\mathbf{z}}$. A pump beam provided by a colimated continuous monochromatic laser can be treated under the paraxial aproximation, being described by a multimode coherent state $\mid v_{\epsilon_p}(\mathbf{q}_p)\rangle$ where $v_{\epsilon_p}(\mathbf{q}_p)$ is the angular spectrum, $\mathbf{q}_p$ is the transverse wave vector and ϵ_p is the pump polarization vector. Furthermore, for a thin and wide crystal the phase matching condition is very restrictive only for the transverse components of pump, signal and idler wave vectors, imposing $\mathbf{q}_s + \mathbf{q}_i = \mathbf{q}_p$.

Now, let us consider the experimental situation found in refs. [1] and [3]. The two crystals employed were cut for type I phase match with optical axis rotated 90^o with respect to each other. We shall suppose that the first crystal is oriented so that it is pumped by the vertical (V) polarization component, generating horizontally (H) polarized signal and idler. The second crystal is therefore pumped by the horizontal (H) polarization component, generating vertically (V) polarized signal and idler. The down converted modes are initially in the vacuum state so that the quantum state generated by parametric interaction up to first order is given by

$$\begin{aligned} \mid \psi\rangle &= \mid vac\rangle + C\int d\mathbf{q}_s \int d\mathbf{q}_i \, v_V(\mathbf{q}_s + \mathbf{q}_i) \mid 1_{\mathbf{q}_s,H} 1_{\mathbf{q}_i,H}\rangle \\ &\quad + e^{-i\phi(\mathbf{q}_s,\mathbf{q}_i,d)} \, v_H(\mathbf{q}_s + \mathbf{q}_i) \mid 1_{\mathbf{q}_s,V} 1_{\mathbf{q}_i,V}\rangle \, . \end{aligned} \tag{1}$$

This quantum state gives rise to quantum images imprinted on the nonlocal two-photon correlations. Image-polarization entanglement arises from the polarization dependence of the pump angular spectrum in the quantum state above.

The coincidence count profile on the detection planes, is given by the normally ordered correlation function

$$C(\mathbf{r}_s, \mathbf{r}_i) = \langle \psi \mid E^{-}(\mathbf{r}_s) E^{-}(\mathbf{r}_i) E^{+}(\mathbf{r}_i) E^{+}(\mathbf{r}_s) \mid \psi \rangle \tag{2}$$

where $E^{+(-)}(\mathbf{r})$ is the positive (negative) frequency part of the electric field operator on the detectors. We now take the multimode decomposition of the detected field operator

$$\begin{aligned} E^{+}(\mathbf{r}_l) &= \sin\theta_l \int d\mathbf{q}_l'\, a_H(\mathbf{q}_l')\, e^{i\left(\mathbf{q}_l' \cdot \rho_l + \sqrt{k_l^2 - q_l'^2}\, r_{1l}\right)} \\ &+ \cos\theta_l \int d\mathbf{q}_l'\, a_V(\mathbf{q}_l')\, e^{i\left(\mathbf{q}_l' \cdot \rho_l + \sqrt{k_l^2 - q_l'^2}\, r_{2l}\right)} \,, \end{aligned} \tag{3}$$

where θ_l is the angle between the vertical axis and the transmission axis of the PBS placed in front of detector l ($l = s, i$), ρ_l is the transverse position of detector l and r_{jl} is the distance between the center of crystal j ($j = 1, 2$) and the origin of detection plane l. Moreover, $a_\epsilon(\mathbf{q}_l)$ is the anihilation operator for photons with polarization ϵ (H or V) and transverse momentum $\mathbf{q}_l$. We recall that H polarized photons are created in crystal 1 while V polarized photons are created in crystal 2.

Let us assume for simplicity the degenerate case $k_s = k_i = k$ and $k_p = 2k$, with a symmetric configuration such that the crystals are equidistant from signal and idler detectors: $r_{1i} = r_{1s} = z$ and $r_{2i} = r_{2s} = z'$. Let us also assume the same orientation for the PBS in front of both detectors, so that $\theta_s = \theta_i = \theta_A$. The resulting coindence profile is therefore

$$C(\rho_+) = \frac{\mid C \mid^2}{16} \mid \sin^2\theta_A\, W_V(\rho_+) + \cos^2\theta_A\, W_H(\rho_+) e^{-i\varphi} \mid^2 \,, \tag{4}$$

where $\varphi = 2k\mathbf{d} \cdot \rho_+ / z$, $\rho_+ = \rho_i + \rho_s$, and ρ_l is a transverse position vector at detection plane $l = s, i$. This phase factor φ is responsible for the position interference measured in ref.[3]. Eq.(4) shows that the spacial profile of the coincidence counts is a mixture of the images carried by the two polarization components of the pump beam. An interesting application arises if the two orthogonal polarizations of the pump are rotated by an angle θ_P with respect to H and V. In this case the coincidence profile for two adjacent crystals ($\varphi = 0$) is

$$\begin{aligned} C(\rho_+) &= \frac{\mid C \mid^2}{16} \mid W_y(\rho_+)\,(\cos^2\theta_A \sin\theta_P + \sin^2\theta_A \cos\theta_P) \\ &+ W_x(\rho_+)\,(\cos^2\theta_A \cos\theta_P - \sin^2\theta_A \sin\theta_P) \mid^2 \,, \end{aligned} \tag{5}$$

where x and y are the two orthogonal polarizations of the pump. Let us suppose that W_y carries some information while W_x carries an encoding noisy image. From the result above one can see that for a given θ_P there is always a suitable value of θ_A which filters the information leaving the noisy contribution out of the coincidence profile. This suggests a scheme for criptographic communication where the sender mixes the two images up and sends the twin beams to a receiver who makes the polarization analysis.

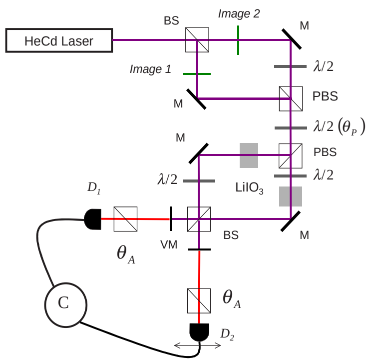

Figure 1: Experimental setup.

Due to the phase matching conditions of our crystals (two $LiIO_3$ crystals cut for type I phase match) it was not possible to employ the two crystals in sequence as in refs.[2] and [3]. We therefore built the experimental setup sketched in Fig.1. The pump beam is divided in a 50%/50% nonpolarizing beam splitter (BS) and an image is formed on each output. The polarization of the beam carrying image 2 is rotated at 90^o by a halfwave plate (HWP) and the two images are recombined on a polarizing beam splitter. Therefore, the pump beam carries two images on orthogonal polarizations, a horizontal (H) and a vertical one (V). The vertical image is a collimated Gaussian and the horizontal image is a displaced focussed Gaussian (Fig.2). We shall use the language suggested by our application proposal and call the two images carried by the pump beam as noise and information, respectively.

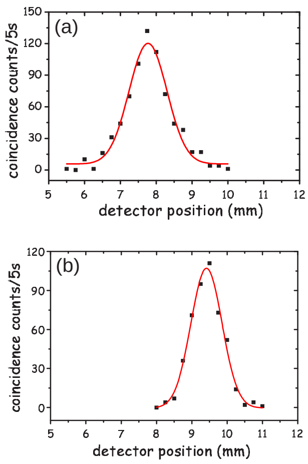

Figure 2: (a) Vertical image transfered to both crystals(noise); (b) Horizontal image transfered to both crystals(information). $\theta_A = 45^o$. Solid lines correspond to gaussian fit.

At the output of the first polarizing beam splitter (PBS), we placed a second HWP to produce a pair of orthogonal polarizations rotated by θ_P with respect to H and V. The second PBS, then, projects images 1 and 2 along H and V, and distributes them between the two crystals. Instead of employing two crystals with orthogonal optical axis, we oriented the optical axis of the two crystals parallel to each other and introduced a HWP before one of them. In order to achieve polarization entanglement another HWP was introduced after the other crystal. Therefore, the quantum state generated by the setup is

$$\begin{aligned} \mid \psi\rangle &= \mid vac\rangle + C \int d\mathbf{q}_s \int d\mathbf{q}_i \, v_H(\mathbf{q}_s + \mathbf{q}_i) \mid 1_{\mathbf{q}_s,H} 1_{\mathbf{q}_i,H}\rangle \\ &\quad + e^{-i\phi(\mathbf{q}_s,\mathbf{q}_i,d)} \, v_V(\mathbf{q}_s + \mathbf{q}_i) \mid 1_{\mathbf{q}_s,V} 1_{\mathbf{q}_i,V}\rangle \, . \end{aligned} \tag{6}$$

Notice that this state is slightly different from the one given by Eq.1. However, this difference is not essential since the entanglement between polarization and transverse momentum degrees of freedom is still present.

Finally, the H polarized and the V polarized twin photons are mixed at a 50%/50% nonpolarizing beam splitter (BS). Each output of this BS is filtered in order to block the transmitted pump beam and sent to a photocounter for coincidence detection. A polarizer is placed before each photocounter, both oriented at an angle θ_A with respect to V. A straightforward calculation on the same lines of the one leading to Eq.(15) gives the coincidence profile as

$$\begin{aligned} C(\rho_+) &= \frac{\mid C \mid^2}{16} \mid W_y \, (\sin^2\theta_A \sin\theta_P + \cos^2\theta_A \cos\theta_P) \\ &+ \; W_x \, (\sin^2\theta_A \cos\theta_P - \cos^2\theta_A \sin\theta_P) \mid^2 , \end{aligned} \tag{7}$$

where $W_x(\rho)$ and $W_y(\rho)$ are the two images carried by the pump beam, with polarizations x and y rotated by θ_P with respect to H and V. As before, for a given orientation of the pump (θ_P), there is a orientation of the analizers (θ_A) which filters the noise and provides the cleanned information.

This principle is experimentally demonstrated in Fig.3, where θ_P is fixed at -30^o while the analizers are varied to provide the noiseless information. In Fig.3(a) the analizers are oriented at 45^o giving a noisy information while in Fig.3(b) the niseless information is recovered when the analizers are set to $\theta_A = 37^o$.

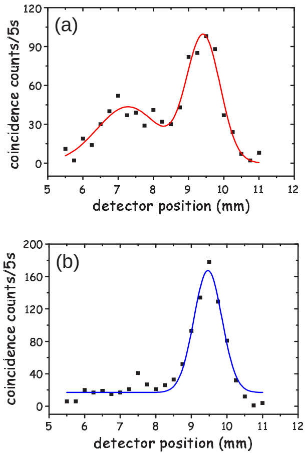

Figure 3: (a) Information in the presence of noise. $\theta_P = -30^o$ and $\theta_A = 45^o$; (b) Recovered information. $\theta_P = -30^o$ and $\theta_A = 37^o$. Solid lines correspond to gaussian fit.

In summary, we applied a multimode theory for parametric down conversion with two crystals. Besides the usual polarization entanglement, this setup is shown to be a device for image-polarization entanglement and potential applications on quantum communication can be envisaged. Our experimental results are in very good agreement with the theory developed.

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