Hole burning in the photon-number distribution of travelling field states

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Abstract

We present a scheme to prepare states of the quantized electromagnetic field possessing holes in the photon-number distribution at controlled positions for travelling fields. The procedure extends to running optical modes the method for production of such states in a microwave cavity.

Preparation of specific nonclassical states of the electromagnetic field turns out to be important for several studies in Quantum Optics. For example, the procedure to generate Schrödinger-cat states (superpositions of mesoscopic coherent states) [1] and their higher-order generations [2] has been extended to create, inside a microwave high-Q cavity, states possessing holes in their photon-number distributions (PND) located at desired positions [3], starting from a coherent state. This scheme involves, after the ejection of two-level Rydberg atoms with defined velocity, an intensity-controlled Ramsey zone which fixes the atomic superpositions the atoms enter the high-Q cavity (having initially a coherent field state), where dispersive atom-field interaction takes place, and a second $(\pi/2)$-pulse Ramsey zone before selective atomic detection. This process generates a state with one hole in the PND, and sequential passage of other atoms, with appropriated changes in the intensity of the first Ramsey zone, leads to states possessing M holes in the PND. Such states may be useful for optical storage and communication.

In the present work we address the question of generating states with holes in the PND in running modes of optical fields. We begin considering the production of travelling states having one hole in the PND located at a desired position. The starting point is the proposal of production of optical Schrödinger-cat states presented in Ref. [4], also used to create (approximately) a hole in the middle of the PND from an intense coherent state [5]. The modified scheme is presented in Fig. 1(A).

A Mach-Zehnder interferometer (MZI), fed by a single photon and the vacuum state in the two ports, contains a Kerr medium in one arm, which couples an external mode (**a**) (being initially in a coherent state) to the **b**-mode of the MZI, and a controllable phase shifter in the other arm (mode **c**). The beam splitters have the reflectance equal to the transmittance, that is $|\Psi_{out}\rangle_{bc} = \exp\left[i\frac{\pi}{4}\left(\hat{b}^\dagger\hat{c} + \hat{b}\hat{c}^\dagger\right)\right]|\Psi_{in}\rangle_{bc}$, so that after BS1 the state of the system is

$$\frac{1}{\sqrt{2}}\left(|1\rangle_b|0\rangle_c + i\,|0\rangle_b|1\rangle_c\right)|\alpha\rangle\ . \tag{1}$$

The dispersive Kerr interaction between modes **a** and **b** can be approximately described by the Hamiltonian

$$\hat{H}_K = \hbar K\,\hat{a}^\dagger a\,\hat{b}^\dagger\hat{b}\ , \tag{2}$$

where K is proportional to the third-order nonlinear susceptibility of the medium. On the other hand, the phase shifter is supposed to add a phase $e^{i\theta}$ to the field passing by it. Thus, just before the second beam splitter (BS2), the system state is

$$\frac{1}{\sqrt{2}}\left(e^{i\theta}\ |1\rangle_b|0\rangle_c|\alpha\rangle + i\ |0\rangle_b|1\rangle_c\left|\alpha e^{-i\varphi}\right\rangle\right) \tag{3}$$

where $\varphi = Kl/v$, l being the length of the Kerr medium and v the velocity of light in it. The BS2, which further entangles the modes through the transformation

$$|1\rangle_b|0\rangle_c \quad\longrightarrow\quad \frac{1}{\sqrt{2}}\left(|1\rangle_b|0\rangle_c + i\ |0\rangle_b|1\rangle_c\right) \tag{4}$$

$$|0\rangle_b|1\rangle_c \quad\longrightarrow\quad \frac{1}{\sqrt{2}}\left(|0\rangle_b|1\rangle_c + i\ |1\rangle_b|0\rangle_c\right)\ , \tag{5}$$

and the state of the system becomes

$$\frac{1}{2}\left[|1\rangle_b |0\rangle_c \left(\left|\alpha e^{-i\varphi}\right\rangle - e^{i\theta}\ |\alpha\rangle\right) + i\ |0\rangle_b |1\rangle_c \left(\left|\alpha e^{-i\varphi}\right\rangle + e^{i\theta}\ |\alpha\rangle\right)\right]\ . \tag{6}$$

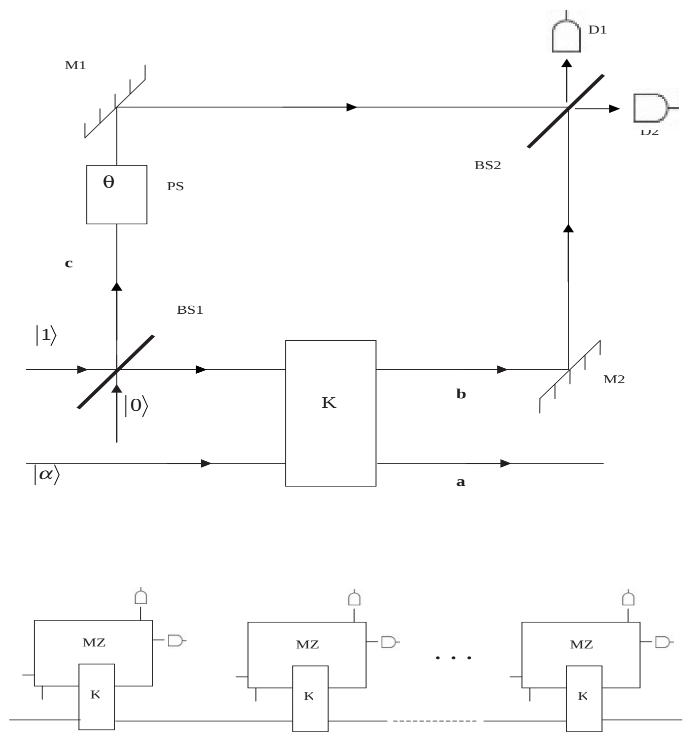

Figure 1: (A) Experimental setup to produce a state with a hole in the PND at a chosen position for travelling modes (upper scheme); (B) Schematic illustration of the sequence of MZI used to produce states with more than one hole (lower part).

Now, if the detector D1 (D2) catches a photon, corresponding to the detection of the state $|1\rangle_b |0\rangle_c$ ($|0\rangle_b |1\rangle_c$), then the **a**-mode is projected into the states

$$\begin{aligned} |\Psi\rangle_j &= \mathcal{N}_j \left(|\alpha\rangle + (-1)^j e^{-i\theta} \left|\alpha e^{-i\varphi}\right\rangle\right) \\ &= \mathcal{N}_j e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \left[1 + (-1)^j e^{-i(\theta + n\varphi)}\right] |n\rangle\ , \end{aligned} \tag{7}$$

where $\mathcal{N}_j$ $(j = 1, 2)$ are normalization constants. We see that, by appropriated choices of the parameters θ and φ (dictated by the phase shifter and the Kerr medium), these states may present a hole in the PND at a desired position. In fact it is found that, if one takes

$$\varphi = \frac{\pi}{N} \quad , \quad \theta = \left(1 - \frac{N_1}{N}\right)\pi\ , \tag{8}$$

where N is an integer large compared with $|\alpha|^2$ and $N_1 \ll N$ is an integer within the range of values of n for which $P_n = |\langle n|\Psi\rangle|^2$ is appreciable, the resulting $|\Psi\rangle_2$ state will possess a hole in the PND for $n = N_1$. Notice that other holes occurs for n belonging to the set $\{N_1 + 2N,\ N_1 + 4N,\ \ldots\}$, but these lie far way from the peak of the PND (for N large enough) where it effectively vanishes [6]. The state $|\Psi\rangle_1$, obtained when the detector D1 fires, will have a hole located at $n = N_1$ if one takes $\varphi = \pi/N$, as before, but adjust the phase shifter to have $\theta = -N_1\pi/N$; however, when the parameters (8) are taken the state $|\Psi\rangle_1$ does not have any hole. From now on we shall work only with states of the kind of $|\Psi\rangle_2$.

It is worth mention that, in our procedure, N is taken very large which implies that φ is small; such small values of φ are the ones likely to occur in practical situations, with a Kerr medium of short length having a currently available nonlinearity. Also, the state possessing one hole obtained in Ref. [5] corresponds to $|\Psi\rangle_2$ with $\theta = 0$ and $\varphi = \pi/|\alpha|^2$ and, in this case, for n equal to the nearest integer to $|\alpha|^2$ one has $P_n \approx 0$; notice that such a hole is only "exact" if $|\alpha|^2$ is an integer.

To create states having more than one hole in the PND one has to consider a sequence of MZI of the type discussed before, with the **a**-mode of each MZI (from the second ahead) being fed by the state emerging from the interferometer just behind it, appropriately controlling the phase shifter in each element of the sequence. Such a setup is schematically drawn in Fig. 1(B).

In fact, consider M aligned MZI, the **a**-mode of the first one fed with the coherent state $|\alpha\rangle$, having phase shifter adjusted such that

$$\theta_r = \left(1 - \frac{N_r}{N}\right)\pi \quad , \quad r = 1, 2, \ldots M \ , \tag{9}$$

and all with the same Kerr medium (that is, all leading to $\varphi = \pi/N$, for N large compared with $|\alpha|^2$). If only the detectors in the horizontal output ports of the interferometers detect photons, then the state emerging from the M-th MZI is given by

$$|\psi\rangle_M = \mathcal{N}_M \, e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \prod_{r=1}^{M} \left(1 + e^{-i[1+(n-N_r)/N]\pi}\right) |n\rangle \ . \tag{10}$$

Now, if the N_r ($r = 1, 2, \ldots M$) are all distinct integers much smaller than N , then the above state possesses M holes in its PND located at $n \in \{N_1, \ldots, N_M\}$. Although the parameter N does not interfere in the position of the holes, it influences the height of the hills between the holes in the PND. The nature of the photon statistics, for states having the same number of holes and the same $|\alpha|$, depends on the position of the holes; as expected, sub-Poissonian statistics prevails when the PND is more concentrated in large values of n.

Finally, we point out that the quantum state engineering (QSE) of such states in a microwave cavity and their QSE for running optical modes can be mapped into one another. In fact, one sees that the atom-field dispersive interaction in the cavity corresponds to the dispersive Kerr interaction between the external mode and the photon entering the interferometer when they cross the medium. Also, the first beam splitter plus the phase shifter in the **c**-arm of the MZI plays the role of the controlled first Ramsey zone in the cavity scheme, while the second beam splitter corresponds to the second Ramsey zone. While in the cavity one has entangled atom-field states with the desired state being produced after the selective atomic detection, for travelling fields one deals with entangled states of the MZI and the external mode and the hole burned state is obtained after the selective detection of the photon in the output ports of the interferometer.

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