Topological defects on moiré fringes of spiral zone plates

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Abstract

We present a study of spatial structures created by superposition of spiral zone plates used for generating optical beams with phase singularities. Moiré fringes are observed showing topological defects similar to those appearing in interference patterns of optical vortices. A brief theoretical discussion is included supporting the similarities between both phenomena. Our results may lead to interesting applications to digital information processing by optical means.

Recently, a great deal of work has been devoted to the study of light beams carrying phase singularities [1], also called optical vortices. Both quantum and classical aspects have been addressed in the literature. Transference of orbital angular momentum to matter [2], parametric down-conversion with optical vortices, both in the spontaneous [3] and stimulated [4] cases are some studies with optical vortices. Examples of optical vortices are the Laguerre-Gaussian (LG) modes which are solutions of the paraxial wave equation in cylindrical coordinates [5]. In these modes the phase depends on the azimuthal coordinate so that a $2m\pi$ ($m = \pm1, \pm2, ...$) variation of the phase occurs in a 2π rotation around the propagation axis, where m is the topological charge.

The available methods for generating optical vortices are essentially based either on holographic methods [6] or on astigmatic mode conversion [7]. In the latter, a TEM_{01} Hermite-Gaussian (HG) mode is incident on a pair of cylindrical lenses suitably placed in order to transform the input mode into an LG vortex mode. The holographic method has some advantages for its simplicity while astigmatic mode converters allow for low losses and are more adequate when power is essential. The holograms used for vortex generation are a variation of the well known Fresnel zone plates (FZP) (fig.(1a)) frequently used either as an amplitude or a phase mask for focussing. Besides the usual focussing effect, spiral zone plates (SZP) such as those shown in figs.(1b) and (1c) can generate optical vortices. The number and the sense of the dark (or white) spirals determine the topological charge of the vortex.

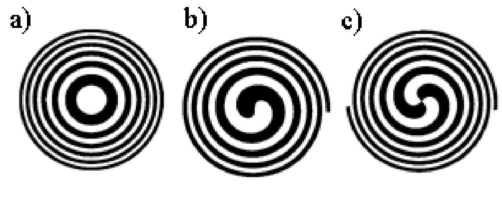

Figure 1: Zone plates for a-) $m = 0$ (Fresnel zone plate), b-) $m = 1$ and c-)$m = 2$.

An incident monochromatic light beam diffracted by such a mask converges to its focus and a confocal lens recollimates the beam already in the vortex mode. The presence of phase singularity is evidenced both from doughhnut shape intensity profile and from interference patterns,where the vortices signatures appears as topological defects or dislocations on the interference fringes.

In this work we study the spatial properties of holograms used for genereting optical vortices. It is well known that superposition of two spatially repetitive structures (not necessarrily periodic) gives rise to news structures sometimes not contained in either of the superposed ones, leading to the so called moiré fringes [8] with many aplication in differents fields [9]. Moiré fringes are weel known for a variety of repetitive structures including the FZP, for wich they appear as a sequence of parallel straight lines. We have studied the moiré fringes generated by superposition of two SZP and topological defects are observed similar to vortex signature on interference fringes. We also present a theorical treatment based on ref.[10], wich suports our experimental results. A Fourier analysis of the repetitive structures being superposed is performed from which the resulting moiré patterns can be extracted.

When the structures are superposed, each layer is represented by a transmitance function $t_1(r,\phi)$ assuming values from 0 (black) to 1 (white), written in this case in polar coordinates. The superposition is then described by the product of individual layers:

$$t(r,\phi) = t_1(r,\phi)\, t_2(r,\phi) \; . \tag{1}$$

The SPZ studied here belongs to the class of coordinate transformed structures discribed in ref.[10]. In this case the transmitance functions can be written as $t_i(r,\phi) = p(x')$ where p is some periodic function and $x' = g_i(r,\phi)$ represents the coordinate transformation. A general expression for the zone plates studied here is

$$g(r,\phi) = \alpha\, r^2 + m\,\phi \; , \tag{2}$$

where α gives the radial scale and m is the angular order which determines the topological charge for vortex generation. The value $m = 0$ reproduces the usual Fresnel zone plate. By taking the Fourier expansion for $p(x')$ and writing the transmittance functions as

$$t_i(r,\phi) = \sum_{n=-\infty}^{\infty} c_n^{(i)} \exp[2i\pi n\, g_i(r,\phi)] \; , \tag{3}$$

we obtain the Fourier expansion for the superposed structures:

$$\begin{aligned} t_1(r,\phi) t_2(r,\phi) &= \sum_{n,l=-\infty}^{\infty} c_n^{(1)} c_l^{(2)} \exp[2i\pi(n\, g_1(r,\phi) \\ &+ \; l\, g_2(r,\phi))] \; . \end{aligned} \tag{4}$$

The moiré fringes can be extracted from the expansion above as the partial sums:

$$f_{k_1,k_2}(r,\phi) = \sum_{m=-\infty}^{\infty} c_{mk_1}^{(1)} c_{mk_2}^{(2)} \exp[2i\pi m\, g_{k_1,k_2}(r,\phi)] \; , \tag{5}$$

where k_1 and k_2 are coprime integers and $g_{k_1,k_2}(r,\phi) = k_1\, g_1(r,\phi) + k_2\, g_2(r,\phi)$ is the geometric layout of the moiré structure. The structure f_{k_1,k_2} is not present in any of the orginal ones but appears in the superposition pattern. Usually, only the lowest order moirés fall in the visible frequency range and therefore we shall focus on the $f_{1,-1}$ contribution. Let us first consider the superposition of two SZP with topological charges m_1 and m_2, slightly displaced along x such that

$$\begin{aligned} g_1(r,\phi) &= \alpha((x-\epsilon/2)^2 + y^2) + m_1\phi \\ g_2(r,\phi) &= \alpha((x+\epsilon/2)^2 + y^2) + m_2\phi \end{aligned} \tag{6}$$

with $\epsilon \ll 1$. Therefore, keeping terms up to first order in ϵ, we obtain

$$g_{1,-1}(r,\phi) = 2\,\alpha\,\epsilon\, x + \Delta m\, \phi \tag{7}$$

where $\Delta m = m_1 - m_2$. The resulting geometric layout is a set of curves representing dark and white fringes: $g_{1,-1}(r,\phi) = q\,\pi$ $(q = 0, \pm 1, ...)$. For the usual case of FZP $(m1 = m2 = 0)$ one obtains a sequence of straight lines $2\,\alpha\,\epsilon\, x = q\,\pi$, regularly spaced. The same layout is obtained when two SZP with $m_1 = m_2$ are superposed. For $m_1 \neq m_2$, $g_{1,-1}$ becomes dependent on ϕ, which is ill defined at the origin. Therefore, a topological defect appears on the geometric layout of the moiré fringes. In practical terms, Δm bifurcations appear in the geometric layout, in analogy with the interference patterns of optical vortices. In the right column of fig.(2) we present the moiré fringes obtained by superposition of two SZPs. One of them was placed on a X-Y stage with micrometer control to produce small displacements ϵ along the X-direction. The superposed SZPs were illuminated by white light and the resulting image with moiré fringes was captured by a CCD camera. In the left column, the analogous interference patterns between optical vortices are shown. In fig.(2a) we present the self interference pattern for an $m = 1$ optical vortex, produced by diffraction of a 7mW He-Ne laser beam (λ =632.8nm) by a SZP with $m = 1$. The optical vortex traverses a slightly misaligned Mach-Zender interferometer whose output was captured by a CCD camera . The corresponding moiré fringes between two $m = 1$ masks is shown in fig.(2d). The similarity between the interference pattern and the corresponding moiré fringes is very clear. In both cases straight lines appear.

For the result presented in fig.(2b), an $m = 0$ beam was sent into the interferometer and the optical vortex was produced in one arm , thus providing the intereference between $m = 1$ and $m = 0$. On the other hand, in fig(2e) the moiré pattern resulting from the superposition of a FZP ($m = 0$) and a SZP with $m = 1$ is shown. Again, the same topological defect arises in both cases, corresponding to $\Delta m = 1$. However, a curved bifurcation appears in fig.(2e) which is due to a small difference in the radial scale between the two zone plates, that motivated us investigate moié patterns of SPZs with different radial scale.

In fig.(2c), the interference pattern between $m = 1$ and $m = -1$ optical vortices is shown. To provide the $m = -1$ beam, a Dove prism (DP) was inserted in one arm of the interferometer . The analogous moiré fringes are shown in fig.(2f) and a topological defect corresponding to $\Delta m = 2$ is clear in both cases.

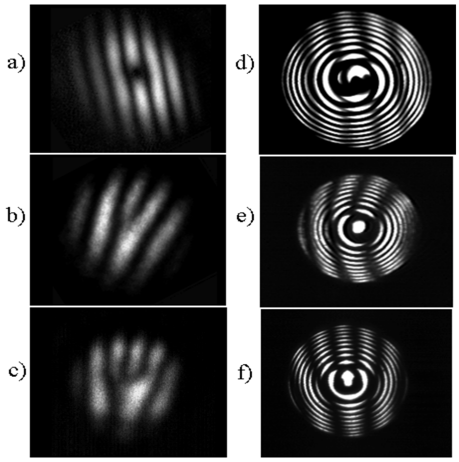

Figure 2: Interference patterns between two optical vortices with: a-) $m_1 = m_2 = 1$, b-) $m_1 = 1$ and $m_2 = 0$, c-) $m_1 = 1$ and $m_2 = -1$; and analogous moiré fringes for superposition of two spiral zone plates: d-) $m_1 = m_2 = 1$, e-) $m_1 = 1$ and $m_2 = 0$, f-) $m_1 = 1$ and $m_2 = -1$.

In the case of different radial scale, presented above, we have:

$$g_{1,-1}(r, \phi) = \Delta\alpha\, r^2 + \Delta m\, \phi \ , \tag{8}$$

which means that the resulting structure is again a SZP with topological charge $\Delta m = m_1 - m_2$ and radial scale $\Delta\alpha = \alpha_1 - \alpha_2$. In Fig(3) we present our experimental results in agreement with the theoretical predictions above. An interference patter analogy applies here also, but in this case the interference is between two divergent optical vortices.

In summary, we have demonstrated the appearance of topological defects in moiré fringes obtained from superposition of SZPs. These defects are analogous to those typical of interference patterns with optical vortices. Actually, the SZP studied here are a special class of such structures. The arithmetics that governs the topological charge of the moiré structure in terms of the original ones may lead to interesting applications to digital information processing by optical means.

One may envisage, for example, an interesting application of the moiré effect with SZP to cryptography. Let us suppose that a digital information, composed by a sequence of integer numbers, is coded on a set of holograms according to some secret rule (the addition of a given random sequence, for example). The coded information would then be another sequence of integer values corresponding to the topological charges in the hologram set. In order to recover the original information one can use a second hologram set, the reading one, such that supperposition of the reading set and the coded one would invert the coding rule yielding the original information through the resulting moiré structures. The original information is therefore given by the topological charges in the moiré sequence, which can be read by optical means.

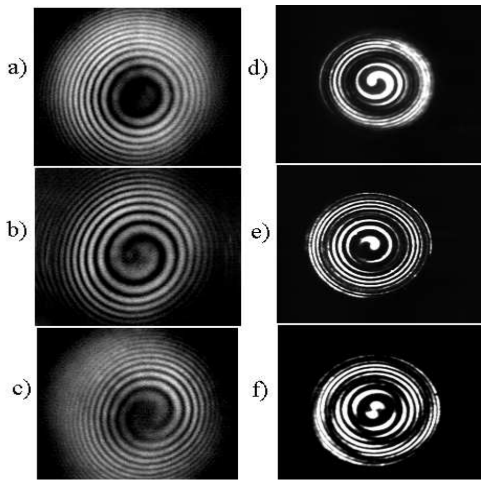

Figure 3: Interference patterns between a divergent optical vortex and a collimated one for: a-) $m_1 = m_2 = 1$, b-) $m_1 = 1$ and $m_2 = 0$, c-) $m_1 = 1$ and $m_2 = -1$; and analogous moiré fringes for spiral zone plates with different radial scales: d-) $m_1 = m_2 = 1$, e-) $m_1 = 1$ and $m_2 = 0$, f-) $m_1 = 1$ and $m_2 = -1$.

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