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Classicality and Anticlassicality Measures of Pure and Mixed Quantum States

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Abstract

We introduce a simple measure of "classicality" of pure and mixed quantum states as a maximum value of the Hilbert-Schmidt "scalar products" between the renormalized operators of the state concerned and all displaced thermal states. Choosing Fock states as the reference set, we introduce the measure of "anticlassicality". Gaussian states are shown to be the closest to thermal states possessing the same degree of the quantum purity. On the contrary, Fock states appear to be more close to mixed thermal states than to pure coherent states

Introduction

Since Glauber's paper [1], quantum states for which the so-called P-distribution [2] is nonpositive or more singular than delta function are called "nonclassical". For the past decades, many authors proposed different quantitative measures of "nonclassicality". It seems impossible to reduce all variety of quantum states in infinite-dimensional Hilbert spaces to some unique parameter. Therefore, different existing approaches should be considered sooner as complementary rather than competitive. Here we follow the direction opened by Hillery [3]. It consists in evaluating some kinds of distances in the Hilbert space between the state concerned and a family of states which are assumed to be "classical". The most simple from the point of view of calculations is the Hilbert-Schmidt distance,

$$d_{HS}^2(\rho,\rho_c) = Tr(\rho-\rho_c)^2 = Tr(\rho^2) + Tr(\rho_c^2) - 2Tr(\rho\rho_c),$$

where ρ is the statistical operator of the quantum state concerned, and ρ_c is related to the reference "classical" state. For a pure quantum state, $\rho = |\Psi\rangle\langle\Psi|$, the reference states are usually identified with coherent state, $\rho_c = |\alpha\rangle\langle\alpha|$, which are assumed to be "the most classical" ones. Then the calculation of distance is reduced to calculating the scalar product $\langle\alpha|\Psi\rangle$. But, coherent states represent only a small subset at the border of the set of all "classical" states. It seems reasonable to enlarge the family of reference states ρ_c, incorporating [4] all displaced thermal states. Unfortunalely, even the Hilbert-Schmidt distance becomes complicated in such a case when one deals with mixed states. Here we introduce a new "classicality measure", which is proportional to the term $Tr(\rho\rho_c)$.

Results and Discussions

Since practically all information about the closeness of ρ and ρ_c is contained in the term $Tr(\rho\rho_c)$, we consider only this term and search for its maximum with respect to the family of "classical" states. Considering $\rho_c = |\alpha\rangle\langle\alpha|$ and $\rho = |\Psi\rangle\langle\Psi|$, we have that the quantity $\max_\alpha |\langle\alpha|\Psi\rangle|^2$ seems a good measure of the "classiaclity", because it equals 1 for coherent states and less than 1 for all other pure state. But, for mixed states, a generalization of scalar product for the pure states is necessary. Thus, we define the "classicality measure" as [5]: $C = \max_{\rho_c} Tr(\rho'\rho'_c)$ where $\rho' \equiv \rho/\sqrt{Tr(\rho^2)}$, obtaining for the pure states $C = \max_{\rho_c}\langle\Psi|\rho'_c|\Psi\rangle$. For the Fock state $|n\rangle$, the quantity $Tr(\rho\rho_c) \equiv f(n,\alpha)$ is reduced to the probability $\langle n|\rho'_c|n\rangle$ of discovering n quanta in the displaced thermal state. We arrive at the function

$$f(\eta,\alpha) = \sqrt{1-\eta^2}\,\eta^n \exp(-|\alpha|^2\sqrt{1-\eta^2})L_n(-|\alpha|^2(1-\eta)^2/\eta),$$

where $L_n(z)$ is the Laguerre polynomial. For $\eta = 0$ we have the Poissonian distribution of the coherent state which has maximum at $|\alpha|^2 = n$. The function $f(\eta,\alpha)$ has maximum at $\eta_n = \sqrt{n^n/(1+n)^{n+1}}$, so that $\tilde{C}_\eta^{(n)} = \sqrt{n^n/(1+n)^{n+1}}$. One can verify that $\tilde{C}_\eta^{(n)} > \tilde{C}_\alpha^{(n)}$ for any $n \ge 1$. The classicality of the Fock state is given by $\tilde{C}_\eta^{(n)}$, and the pure state $|n\rangle$ is more close not to the pure coherent state with $\overline{n_{coh}} \equiv |\alpha|^2 = n$, but to the mixed thermal state with $\bar{n}_{th} \equiv \eta/(1-\eta) = \sqrt{n}(\sqrt{n}+\sqrt{n+1})$. The Fock states are considered as “the most quantum” states and we use them as reference states for the “anticlassicality”: $A_1 = \max_{n\ge1}\langle n|\rho|n\rangle$. As example, consider the squeezed vacuum state. Their photon distribution function is different from zero only for $n = 2m$ and it can be expressed by $p_{2m}^{(sqv)} = (1+\bar{n})^{-1/2}\frac{(2m)!}{(2^m m!)^2}\left(\frac{\bar{n}}{1+\bar{n}}\right)^m$. Therefore, we obtain the dependence $A_1^{(sqv)}(\bar{n}) = \frac{\bar{n}}{2(1+\bar{n})^{3/2}}$. And for the coherent phase state we have $A_1^{(\varepsilon)}(\bar{n}) = \bar{n}(1+\bar{n})^{-2}$. In order to distinguish pure and mixed states having the same diagonal elements in the Fock basis, we multiply the quantity $\langle n|\rho|n\rangle$ by μ, because mixed states are more anticlassical than pure ones. Adopting the definition $\tilde{A}_1 = \mu\max_{n\ge1}\langle n|\rho|n\rangle$, we obtain for the thermal states the expression $\tilde{A}_1^{(th)}(\bar{n}) = \frac{\bar{n}}{(1+\bar{n})^2(1+2\bar{n})}$.

Conclusions

We introduced new measures of “calssicality” and “anticlassicality” of quantum states, wich characterize the “fidelity” of the chosen states to the whole families of states considered as “classical” or “maximally quantum”. We showed that the most close to arbitrary Gaussian states are thermal states, with the same degree of quantum purity $Tr\rho^2$. Also, thermal states are more close to the Fock states than coherent states. The new measures could be used in studies of decoherence processes and for the classification of different families of nonclassical states.

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