Nonlinear displaced number states

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Abstract

The nonlinear displaced number states are defined. The coefficients \( C_m \) of its number state expansion are obtained by a recurrence relation, and the statistical properties of such states are investigated with particular attention to their nonclassical effects.

Introduction

The coherent states \( |\alpha\rangle \) of the electromagnetic field are defined by.

\[
|\alpha\rangle = \hat{D}(\alpha)|0\rangle
\]

(01)

Where \( \hat{D}(\alpha) \) is the displacement Glauber operator\,[1] given by.

\[
\hat{D}(\alpha) = e^{\alpha\hat{a} - \alpha^*\hat{a}^\dagger}.
\]

(02)

These states define the limit between the classical and nonclassical behaviour of the radiation field as far as the nonclassical effects are considered.

A generalization of the coherent state is the called displaced number state \( |n,\alpha\rangle \) which is defined by\,[2].

\[
|n,\alpha\rangle = \hat{D}(\alpha)|n\rangle.
\]

(03)

For \( n = 0 \), the displaced number state, Eq.(03) reduces to the coherent state.

Another generalization of the coherent states is the nonlinear coherent state \( |\alpha, F\rangle \), defined as the eigenstates of the operator \( F(\hat{n})\hat{a} \), where \( F(\hat{n}) \) is an operator-valued function of the number operator \( \hat{n} \)\,[3].

While the coherent states are eigenstates of the annihilation operator \( \hat{a} \), the eigenstates of the operator \( \hat{a}^2 \) are called even and odd coherent states. These states can be written as a combination of the coherent states \( |\alpha\rangle \) and \( |\alpha\rangle \)\,[4]. Recently it was also introduced the even and odd nonlinear coherent states\,[5], which are eigenstates of the operator \( F(\hat{n})\hat{a}^2 \).

Following we will introduce the nonlinear displaced number states, as a generalization of some of the mentioned states of the literature.
Nonlinear displaced number state

The nonlinear displaced number state \( |n, \alpha, F\rangle \) are defined as the eigenstates of the operator \( F(\hat{n})\hat{b} \), where \( F(\hat{n}) \) is an operator-valued function of the number operator \( \hat{n} \) and \( \hat{b} \) is the operator for which the displaced number state, defined in Eq.(03) is an eigenstate, with eigenvalue \( n \). In fact, the operator \( \hat{b} \) comes from a unitary transformation of the number operator \( \hat{n} \).

\[
\hat{b} = \hat{D}(\alpha)\hat{n}\hat{D}(\alpha) = \hat{D}(\alpha)\hat{a}\hat{a}^+\hat{D}(\alpha)
\]

and since \( \hat{D}(\alpha) \) is an unitary operator

\[
\hat{D}(\alpha)\hat{D}^\dagger(\alpha) = \hat{D}^\dagger(\alpha)\hat{D}(\alpha) = 1
\]

we can write

\[
\hat{b} = \hat{D}(\alpha)\hat{a}^+\hat{D}^\dagger(\alpha)\hat{D}(\alpha)\hat{a}\hat{D}^\dagger(\alpha) = \hat{a}^+\hat{a} - \alpha\hat{a}^+ - \alpha^*\hat{a} + |\alpha|^2
\]

It is easy to show that

\[
\hat{b}|n,\alpha\rangle = n|n,\alpha\rangle.
\]

The nonlinear displaced number state satisfy the relation

\[
F(\hat{n})\hat{b}|n,\alpha,F\rangle = n|n,\alpha,F\rangle
\]

Expanding \( |n,\alpha,F\rangle \) in terms of the number state \( |m\rangle \)

\[
|n,\alpha,F\rangle = \sum_{m=0}^{\infty} C_m |m\rangle
\]

and substituting in Eq.(08), after some calculations we find the recurrence formula to the coefficients \( C_m \) of the nonlinear displaced number state given by

\[
C_{m+1} = \frac{\left| F(m + |\alpha|^2) - nF(m) \right|}{\alpha^*F(m)\sqrt{m+1}} C_m
\]

with \( m = 0, 1, 2, 3 \ldots \) and \( C_{-1} = 0 \). As we can see in Eq.(10) all coefficients \( C_m \) can be obtained from the \( C_0 \) coefficient, which can be obtained by the normalization condition of the nonlinear displaced number state

\[
\langle n,\alpha,F|n,\alpha,F\rangle = 1.
\]

All the statistical properties of the nonlinear displaced number state can be investigated when we know its coefficients of the number representation. Some of this statistical properties was investigated, such as photon number distribution and photon statistic. Comparing its properties with the properties of the displaced number state, we have been note that this state has many interesting nonclassical effects.
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References


