Effect on the theoretical cross sections of common approximations assumed in coupled-channel calculations.


What is the reason to study this subject?

Experimental Data x Theoretical Cross Sections

Conclusions about the underlying physics.
\[ {^{16}\text{O} + ^{154}\text{Sm}} \rightarrow 2^+; \ E^* = 0.08 \text{ MeV}; \ \beta = 0.39 \]

Coupled-Channel (CC) = Frozen Approximation (FA)

- CC approach (coupling potentials)
  \[
  V_{nm}(R) = \int \phi_n(s) V(R,s) \phi_m(s) \, ds
  \begin{cases}
  \phi_n \Rightarrow \text{Modeling} \\
  V(R,s) \Rightarrow \text{Numerical Calculation}
  \end{cases}
  \]

- FA formalism.
- Approx. related to nuclear states \( \phi_n \).
- Approx. related to the interaction \( V(R,s) \).
Coupled-Channel Formalism

\[ \hat{H}_S \phi_n(s) = E_n \phi_n(s) \]

\[ V(\vec{R}, s) = V_N(\vec{R}, s) + V_C(\vec{R}, s) \]

Sch. Equation: \[ [\hat{H}_{RS} + \hat{H}_S] \psi(\vec{R}, s) = (E + E_0) \psi(\vec{R}, s) \]

Isocentrifugal Approximation
(neglecting angular momentum transfer)

For each \[ \ell \implies \psi_\ell(\vec{R}, s) = \sum_n \psi_\ell^{(n)}(R) \phi_n(s) \]
\[- \frac{\hbar^2}{2 \mu} \frac{d^2 \psi^{(n)}_{\ell}}{dR^2} + \left[ V_{nn}(R) + \frac{\ell (\ell + 1) \hbar^2}{2 \mu R^2} - iW(R) \right] \psi^{(n)}_{\ell}(R) + \]

\[+ \sum_{m \neq n} V_{nm}(R) \psi^{(m)}_{\ell}(R) = \left( E + E_0 - E_n \right) \psi^{(n)}_{\ell}(R) \]

**Coupling Potentials:** \( V_{nm}(R) = \int \phi_n(s) V(R, s) \phi_m(s) \, ds \)

**Asymptotic Behavior**

\[\psi^{(n)}_{\ell}(R \to \infty) = \frac{i}{2} \left[ \delta_{n0} H^-_{\ell}(R) - S^{(n)}_{\ell} H^+_{\ell}(R) \right] \]

\[H^\pm_{\ell}(R) = G_{\ell}(R) \pm iF_{\ell}(R) \]

**Cross Sections:** \( S^{(n)}_{\ell} \Rightarrow \sigma_n \)
Frozen Approximation

Rotational model \[ \Rightarrow E_I^* = I(I+1)\hbar^2 / 2\mathcal{I} \]

Vibrational model \[ \Rightarrow E_n^* = n\hbar\omega \]

Small \( E^* \Rightarrow \) Large period of vibration (or rotation) in comparison with the collision time.

In each collision \[ \begin{cases} \bar{\mathcal{R}} \text{ varies} \\ \text{deformation } (s, \theta) \text{ is fixed} \end{cases} \]

For each (frozen) deformation \( s \) \[ \Rightarrow \]

\[ S^{(0)}_{\ell}(s) = \int S^{(0)}_{\ell}(s) P(s) \, ds \]

\[ P(s) = |\phi_0(s)|^2 \Rightarrow \text{Probability distribution of the } s \text{ coordinate} \]
The Coupling Scheme

\[ R = R_0 [1 + \beta Y_{20}(\theta)] \]

Nuclear Density:

\[ \rho(\vec{r}) = \frac{\rho_0}{1 + \exp \left( \frac{r - R(\theta)}{a} \right)} \]
\[ V_c = \int\int \frac{dq_1 \ dq_2}{R'} = \int\int \frac{e^2}{|\vec{R} - \vec{r}_1 + \vec{r}_2|} \rho_1(\vec{r}_1) \rho_2(\vec{r}_2) \ d\vec{r}_1 \ d\vec{r}_2 \]

\[ \rho_1(\vec{r}_1) = \frac{\rho_0}{1 + \exp\left(\frac{r_1 - R(\theta_1)}{a}\right)} \]

\[ R(\theta_1) = R_0\left[1 + \beta Y_{20}(\theta_1)\right] \]

\[ V_c(R, \theta, \beta) \approx V_c(R, s) \]

\[ s = \beta \ R_0 \ Y_{20}(\theta) \]
How accurate is this approximation?

\[ V(\vec{R}, s) = V_N(\vec{R}, s) + V_C(\vec{R}, s) \]

Except at extremely large values of \( \beta \), the hypothesis of a \( s \)-dependent potential is reasonably precise.

\[ s = \beta R_0 Y_{20}(\theta) \]
Models for the nuclear states.

1) Rotational Model

\[ \beta = 0.39 \; ; \; P(\theta) = \frac{1}{2} \sin(\theta) \]

\[ s = \beta \; R_0 \; Y_{20}(\theta) ; P(\theta) \Rightarrow P(s) \]

\[ \sigma_s = \sqrt{\langle s^2 \rangle} = \sqrt{\int s^2 P(s) \; ds} = 0.68 \text{ fm} \]

2) Vibrational (s-harmonic)

\[ P(s) = |\phi_0|^2 = \frac{1}{\sqrt{2\pi\sigma_s}} e^{-s^2/2\sigma_s^2} \]

3) Vibrational (\beta-harmonic)

\[ P(\beta) = \frac{1}{\sqrt{2\pi\sigma_\beta}} e^{-s^2/2\sigma_\beta^2} \]

\[ P(\beta) \text{ and } P(\theta) \Rightarrow P(s) \]
Effect on the fusion cross sections of different models for the nuclear states.

We are not interested in data fit!
Our purpose is to study the effect of different approximations.

Significant sensitivity → No sensitivity to the model

\[ \left( V_B = 60.5 \text{ MeV} \right) \]

Data from [8-10]

- UBPM
- rotational
- s-harmonic
- \( \beta \)-harmonic

Why?
Fusion Cross Section \( \Rightarrow T_\ell = \int T_\ell (s) P(s) \, ds \)

\[
\begin{align*}
 s < 0 & \quad s = 0 & \quad s > 0 \\
 \text{(spherical)} & \quad \text{(planar)} & \quad \text{(deviating)}
\end{align*}
\]

\( T_\ell (s) \) for \( \ell = 0 \)

\( T_\ell (s = 0) = \frac{1}{2} \) for \( E = V_B (\beta = 0) \)

\[ P(s) \]

\[ T^{PA} \]

\[ T^{PA} P (\text{fm}^{-1}) \]
Different energies probe different regions of deformation. In many works, parameters related to coupling amplitudes or bare interaction are adjusted by fitting the high energy data, and the theoretical predictions then extended to the low energy region.

Effect on the Quasi-Elastic Cross Sections

Why?
\[ \sigma_{QE} = \int \sigma_{El}(s) P(s) \, ds \]
• Different energies probe different $s$ values.
• Different angular regions probe different $s$ values.
• Different processes probe different $s$ values.

Take care with extrapolations!
Certain data probe quite extreme deformation regions.

Fusion at $E = V_B - 10$ MeV

$$T^{FA}_\ell = \int T_\ell(s) P(s) \, ds$$

$s_{max} \approx 2.2$ fm $\approx 35\%$ $R_0 \approx 3 \sigma$

$6.2$ fm $0.68$ fm

$s_{max}$
• CC calculations usually involve some modeling to the nuclear states, like the rotational or vibrational models.

• On the other hand, real states possess a mixture of rotational and vibrational properties, which certainly include a certain degree of anharmonicity.

• Then, when using these idealized models in data analyses that probe remote regions of deformation, one should not expect a perfect match between data and theoretical cross sections.
In some cases, the fusion and elastic scattering processes are sensitive to a quite asymptotic region of deformation.

Are the current nuclear structure models and calculations precise enough to describe such a remote region?
Effect of approximations assumed in the calculation of the interaction.

\[ V_c(R, \theta, \beta) = \int \int \frac{e^2}{|\vec{R} - \vec{r}_1 + \vec{r}_2|} \rho_1(\vec{r}_1) \rho_2(\vec{r}_2) \, d\vec{r}_1 \, d\vec{r}_2 \]

\[ R(\theta_1) = R_0 \left[ 1 + \beta Y_{20}(\theta_1) \right] \]

\[ \rho_1(\vec{r}_1) = \frac{\rho_0}{1 + \exp \left( \frac{r_1 - R(\theta_1)}{a} \right)} \]

\[ V_c(R, \theta, \beta) \approx V_c(R, s) \approx V_c^{(0)}(R) + \frac{3Z_1Z_2e^2R_0}{5R^3} s \]

Undeformed Potential

\[ s = \beta R_0 Y_{20}(\theta) \]

First Order Correction
How good is the first order approximation?

\[ R = 11 \text{ fm} \approx R_B \]

\[ ^{16}\text{O} + ^{154}\text{Sm} \]

Second-order approximation for the Coulomb potential

B. V. Carlson, L. C. Chamon and L. R. Gasques,

Nuclear Potential

\[ V_N(R, s) \approx V_N^{(0)}(R) + \sum_n (-1)^n \frac{s^n}{n!} \frac{\partial^n V_N^{(0)}}{\partial R^n} \]

\[ V_N(R, s) \approx V_N^{(0)}(R - s) \]

Only the knowledge of the undeformed (\( \beta = 0 \)) nuclear potential, \( V_N^{(0)}(R) \), is necessary to obtain the deformed interaction.
How good are these approximations?

\[ V_N(R, s) \approx V_N^{(0)}(R) + \sum_n (-1)^n \frac{s^n}{n!} \frac{\partial^n V_N^{(0)}}{\partial R^n} \]

\[ V_N(R, s) \approx V_N^{(0)}(R - s) \]
Effect on the Fusion Cross Section
(calculations within the $s$-harmonic model)
Different approximations assumed to calculate the interaction can produce quite different theoretical fusion and elastic scattering cross sections.
Comparison of FA and usual CC results.

- CC calculations performed with the FRESCO code.

  FRESCO is quite versatile, for example, coupling potentials, \(V_{nm}(R)\), read from external file.

- \(^{16}\text{O} + ^{154}\text{Sm} \Rightarrow\) Rotational Model (quadrupole band).

- Bare potential, \(V^{(0)}_{N}(R)\), read from external file.

- First order approximation for the interactions.

- No isocentrifugal approximation (spin).

- Considering also the (small) excitation energies.
The effect of all states is included in the FA.

CC and FA are equivalent in the present case (small $E^*$).
It is possible (and usual) to assume many approximations in CC calculations performed with powerful codes like FRESKO.
Summary, discussion and conclusion.

Why people usually assume approximations in CC calculations?

\[ V_{nm}(R) = \int \phi_n(s) V(R,s) \phi_m(s) \, ds \]

\( \phi_n \Rightarrow \text{Modeling} \)

\( V(R,s) \Rightarrow \text{Numerical Calculation} \)

Some approximations are very subtle, and one can sometimes make an approximation without realizing this.

\[ \rho(\vec{r}) = \frac{\rho_0}{1 + \exp\left(\frac{r - R(\theta)}{a}\right)} \]

\[ R(\theta) = R_0 [1 + \beta Y_{20}(\theta)] \, a(\beta, \theta) ? \]
• Cross sections corresponding to different processes, or different energies, or even different angular regions, probe different regions of deformation. **Take care with extrapolation!**

• Certain cross sections are very sensitive to quite large deformations, and therefore are sensitive on the asymptotic characteristics of the nuclear states. Then, when using idealized models in data analysis that probe such remote regions, one should not expect a perfect match between data and predictions.

• Approximations made to the interaction should work well for small deformation values. If large deformations are involved in the CC calculations, these approximations can result in inaccurate cross sections. **Take care with highly deformed nuclei!**
(Ridiculous) Example

\[ B(E\lambda) \approx \left( \frac{3ZeR_0^\lambda \beta_\lambda}{4\pi} \right)^2 \]

\[ ^{154}\text{Sm} \rightarrow B(E2) = 4.36 \, e^2 b^2 \Rightarrow \beta_2 \approx 0.39 \]

\[ ^{16}\text{O} \rightarrow B(E3) = 0.0015 \, e^2 b^3 \Rightarrow \beta_3 \approx 1.3 \]
• Data with very small cross sections in extreme regions of energy have been obtained with higher and higher precision.

• We suggest that the precision of the current CC calculations of theoretical cross sections might not be comparable to that of the data.

• Nevertheless, the CC formalism is still the most appropriate approach for the description of heavy-ion reactions.

• When comparing data and predictions, one should take into account the approximations involved in the theoretical calculations, in order to avoid erroneous conclusions about the underlying physics.

• The improvement of the precision of CC calculations is currently an important challenge for understanding heavy-ion nuclear reactions.