A UNITARY AND RENORMALIZABLE THEORY OF
THE STANDARD MODEL IN GHOST-FREE
LIGHT-CONE GAUGE

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Abstract

Light-front (LF) quantization in light-cone (LC) gauge is used to construct a unitary and simultaneously renormalizable theory of the Standard Model. The framework derived earlier for QCD is extended to the Glashow, Weinberg, and Salam (GWS) model of electroweak interaction theory. In the LF quantized QCD in the LC gauge the Lorentz condition is automatically satisfied. In the GSW model, with the spontaneous symmetry breaking present, we find that the ’t Hooft condition \( \partial \cdot A = M_\eta \) accompanies the LC gauge condition. The two transverse polarization vectors for the massive vector boson may be chosen to be the same as found in QCD while the linearly independent third one is parallel to the gauge direction. The sum over polarizations in the Standard model, indicated by \( K_{\mu\nu}(k) \), has several simplifying properties similar to that the polarization sum \( D_{\mu\nu}(k) \) in QCD. The framework is ghost-free and the interaction Hamiltonian of electroweak theory can be expressed in a form resembling that of covariant theory, except for additional instantaneous interactions which can be treated systematically. The LF formulation also provides a transparent discussion of the goldstone boson or electroweak equivalence theorem, as the illustrations show.

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1 Introduction

The quantization of relativistic field theory at fixed light-front time $\tau = (t - z/c)/\sqrt{2}$ [1] has become a standard tool for the analysis of nonperturbative problems in quantum field theory as well as in string [2] and $M$-theory [3]. The light-front quantization of gauge theories in light-cone gauge $A^+ = 0$ is explicitly unitarity – there are no ghost degrees of freedom with nonzero $k^+$. Light-front quantization is especially useful for quantum chromodynamics, since it provides a rigorous extension of many-body quantum mechanics to relativistic bound states: the quark, and gluon momenta and spin correlations of a hadron become encoded in the form of universal process-independent, Lorentz-invariant wavefunctions [4]. The light-front quantization of QCD in its Hamiltonian form thus provides an alternative to lattice gauge theory for the computation of nonperturbative quantities such as the spectrum as well as the light-front Fock state wavefunctions of relativistic bound states [5].

The light-front framework for gauge theory in light-cone gauge $n \cdot A = A^+ = 0$ is a severely constrained dynamical theory with many second-class constraints [6, 7]. These can be eliminated by constructing Dirac brackets, and the theory can be quantized canonically by the correspondence principle in terms of a reduced number of independent fields. The commutation relations among the field operators can be found by the Dirac method. For example, the nondynamical projections of the fermion and gauge field can be eliminated using nonlocal constraint equations. For example, the nondynamical projection of the fermion field can be eliminated using a nonlocal constraint equation. The removal of the unphysical components of the fields results in tree-level instantaneous gluon exchange and fermion exchange interaction terms. The interaction Hamiltonian of QCD can be expressed in a form resembling that of covariant theory with three- and four-point interactions, except for the additional instantaneous four-point interactions which can be treated systematically.

We have recently presented a systematic study [8] of LF-quantized gauge theory following the Dirac method and construct the Dyson-Wick S-matrix expansion based on LF-time-ordered products. In our analysis [8] one imposes the light-cone gauge condition as a linear constraint using a Lagrange multiplier, rather than a quadratic form. We then find that the light-front quantized free gauge theory simultaneously satisfies the covariant gauge condition $\partial \cdot A = 0$ as an operator condition as well as the light-cone gauge condition. The resulting Feynman rule for the gauge field propagator in the l.c. gauge is doubly-transverse

$$\langle 0 | T(A^a_\mu(x)A^b_\nu(0)) | 0 \rangle = \frac{i\delta^{ab}}{(2\pi)^4} \int d^4 k \ e^{-ik \cdot x} \frac{D_{\mu\nu}(k)}{k^2 + i\epsilon}$$

where
\[ D_{\mu \nu}(k) = -g_{\mu \nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{(n \cdot k)} - \frac{k^2}{(n \cdot k)^2} n_\mu n_\nu, \quad n^\mu D_{\mu \nu} = k^\mu D_{\mu \nu} = 0, \]

and \( n_\mu \) is the null four-vector, gauge direction. Thus only physical degrees of freedom propagate. The remarkable properties of \( D_{\nu \mu} \) provide much simplification in the computations of loops. In the case of tree graphs, the term proportional to \( n_\mu n_\nu \) cancels against the instantaneous gluon exchange term. The renormalization constants in the non-Abelian theory can be shown to satisfy the identity \( Z_1 = Z_3 \) at one loop order, as expected in a theory with only physical gauge degrees of freedom. The QCD \( \beta \) function computed in the noncovariant light-cone gauge [8] agrees with the conventional result [10, 11]. Dimensional regularization and the Mandelstam-Leibbrandt prescription [12, 13, 14] for light-cone gauge were used to define the Feynman loop integration [15]. Ghosts only appear in association with the Mandelstam-Liebbrandt prescription. There are no Faddeev-Popov or Gupta-Bleuler ghost terms.

In this paper we extend our light-front quantization analysis to the Glashow, Weinberg and Salam (GWS) Standard Model of electroweak interactions based the nonabelian gauge group \( SU(2)_W \times U(1)_Y \) [16]. The spontaneous symmetry breaking occurs on the light-front to give masses to the electroweak gauge bosons as well as the fermions, but without modification of the light-front vacuum [17, 18]. As shown in the appendix, the phase of a zero longitudinal mode of a c-number function \( \omega \) is broken in the light-front formalism, not the symmetry of the vacuum. The resulting quantum field theory of massive vector, scalar, and fermionic particles is at the same time unitary and renormalizable, as well as practical for computations.

In the conventional quantization, the breaking of gauge symmetry the quantization requires gauge-fixing. In the unitary gauge all the Goldstone fields are gauged away, leaving behind only the physical degrees of freedom. The resulting massive gauge field then is described by the Proca propagator where \( D_{\mu \nu}(k) = -g_{\mu \nu} + \frac{k^\mu k^\nu}{M^2} \). Because of the growing momentum dependence of the gauge propagator, the power counting renormalizability of the theory becomes very difficult to verify in this gauge. 't Hooft, however, demonstrated it by inventing the renormalizable \( R_\xi \) gauges rxi, pesk and employing the gauge symmetry preserving dimensional regularization. This framework, however, does requires one to include in the theory the Faddeev-Popov ghost fields, even in the abelian theory, since they couple to physical Higgs field as well. In the LC gauge LF quantized theory framework there are no ghosts to consider, neither in the abelian nor in the nonabelian case. The massive gauge field propagator has good asymptotic behavior, the massive would-be Goldstone fields can be taken as physical degrees of freedom, and the proof of renormalizability becomes straightforward. We thus obtain the simultaneous realization of unitary and renormalizable gauges.

As a first illustration of our analysis, the LF quantization of a spontaneously
broken abelian Higgs model in LC gauge is discussed in Sec. 2. The propagators of the massive vector boson, the would-be Goldstone field, and the Higgs boson are derived. The polarization vectors of the gauge field, which are all physical, are constructed and their simplifying properties discussed in detail along with the interaction Hamiltonian.

The LF quantization of the electroweak Standard Model is considered in Sec. 3. The LF Hamiltonian framework is constructed following the Dirac method [6, 7] which allows one to self-consistently identify the independent fields and their commutation relations in the presence of the LC gauge condition and other constraints. In the case of QCD, the LF quantized gluon fields satisfy the Lorentz condition $\partial \cdot A = 0$ as an operator condition together with the LC gauge condition. In the case of massive gauge field we find instead the 't Hooft condition, $\partial \cdot A = M \eta$, where $\eta$ is the would-be Goldstone field which acquires mass $M$ together with the corresponding gauge field $A_\mu$. The Fourier transform of the free theory gauge field and its propagator in momentum space then follow straightforwardly. The LF formulation thus provides a transparent discussion of the goldstone boson or electroweak equivalence theorem [21].

The removal of the unphysical components of the fields results [8, 4] in tree-level instantaneous interaction terms which can be evaluated systematically. The interaction Hamiltonian is constructed in Sec. 3 where we restore in the expression the dependent components $A_+^\mu$ and $\psi_-$. It then takes a form close to that of covariant gauge theory without ghost terms, plus instantaneous interactions which are straightforward to handle in the Dyson-Wick expansion constructed in the LF quantized theory.

The Goldstone boson or electroweak equivalence theorem [21] becomes transparent in our formulation as will be illustrated by the computation of some Higgs bosons and top decays in Sec. 4. The computation of muon decay shows the relevance of the instantaneous interactions for recovering the manifest Lorentz invariance in the physical gauge [9] theory framework constructed.

A new aspect of the LF quantization, is that the third polarization of the quantized massive vector field $A^\mu$ with four momentum $k^\mu$ has the form $E^{(3)}_\mu = n_\mu M / n \cdot k$. Since $n^2 = 0$, this polarization vector has zero norm. However, when one includes the constrained interactions of the Goldstone particle, the effective longitudinal polarization vector of the vector particle is $E^{(3)}_{\text{eff} \mu} = E^{(3)}_\mu - k_\mu k \cdot E^{(3)} / k^2$ which is identical to the usual polarization vector of a massive vector with norm $E^{(3)}_{\text{eff} \mu} \cdot E^{(3)}_{\text{eff} \mu} = -1$. Thus, unlike the usual quantization of the Standard Model, the Goldstone particle only provides part of the physical longitudinal mode of the electroweak particles.
2 The Quantization of the Abelian Higgs model in LC Gauge

The mechanism for spontaneous symmetry breaking (SSB) and the tree level Higgs mechanism on the LF has been understood for some time [17, 18, 22]. A review given in Appendix A. The differences between front form theory in the presence of SSB compared with the conventional treatment become apparent even in the abelian gauge theory. The theory is described by

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi^\dagger \phi) \]  

(2)

where \( D_\mu = (\partial_\mu + ie A_\mu) \). We separate the bosonic condensate, <0| \( \phi | \phi > = v/\sqrt{2} \),

\[ \phi(x) = \frac{1}{\sqrt{2}} v + \varphi = \frac{1}{\sqrt{2}} ([v + h(x)] + i\eta(x)) \]  

(3)

such that the real fields \( h(x) \) and \( \eta(x) \) carry vanishing vacuum expectation values. The Lagrangian is rewritten as

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} M^2 A_\mu A^\mu + \frac{1}{2} (\partial_\mu \eta)^2 + M A_\mu A^\mu - \frac{1}{2} (\partial_\mu h)^2 - \frac{1}{2} m_h^2 h^2 \]

\[ + e (h \partial_\mu \eta - \eta \partial_\mu h) A^\mu + e M A_\mu A^\mu h + \frac{e^2}{2} (h^2 + \eta^2) A_\mu A^\mu \]

\[ - \frac{e m_h^2}{2 M} (\eta^2 + h^2) h - \frac{\lambda}{4} (\eta^2 + h^2)^2 + \text{const.} \]  

(4)

where \( m_h^2 = 2 \lambda v^2 = -2 \mu^2 \) is the physical squared mass of the Higgs field \( h(x) \), \( M = e v, 2 \lambda v = e m_h^2 / M, \) and \( 2 \lambda = e^2 m_h^2 / M^2 \). We note the presence of the mixed bilinear term involving the Goldstone field \( \eta \) and the gauge field.

In view of the underlying local \( U(1) \) gauge symmetry one possible choice of the gauge may be taken to be such that the Goldstone mode \( \eta \) is eliminated, the so called “unitary (or unitarity) gauge”, where only the physical fields appear in the Lagrangian. The gauge field is massive and the its (Proca) propagator falls off more slowly than \( 1/k^2 \) for large \( k \). The perturbation theory renormalizability in this gauge is then not possible to demonstrate simply. The alternative of “renormalizability” or \( R_\xi \) gauges were introduced by ’t Hooft [19] where the gauge-fixing term is assumed to be \( \mathcal{L}_{GF} = -(\partial \cdot A - \xi M \eta)^2 / (2\xi) \). The bilinear mixing of \( \eta \) and \( A_\mu \) is then eliminated and for any finite value of \( \xi \) all of the propagators in this class of gauges fall off as \( 1/k^2 \). The theory may also be shown [23] to be perturbatively renormalizable. We note, however, that in the Faddeev-Popov quantization procedure we are required to introduce also the auxiliary ghost fields in the theory with the corresponding piece in the Lagrangian \( \mathcal{L}_{\text{Ghost}} = \bar{c}[-\partial^2 - \xi M^2 (1 + h/v)]c \), which contains the coupling of ghost
fields with the physical Higgs field. In the nonabelian theory there are, in addition, the coupling of ghosts with the gauge field, resulting from the term $c^a(-\partial \cdot D)_{ab}c^b$.

In what follows we will quantize the front form theory described by the Lagrangian (3) in the LC gauge where the ghost fields are seen to decouple in both the nonabelian and abelian theories. The LF coordinates are defined as $x^\mu = (x^+ = x_- = (x^0 + x^3)/\sqrt{2}, x^- = x_+ = (x^0 - x^3)/\sqrt{2}, x^\perp)$, where $x^\perp = (x^1, x^2) = (-x_1, -x_2)$ are the transverse coordinates and $\mu = -, +, 1, 2$. The coordinate $x^+ \equiv \tau$ will be taken as the LF time, while $x^-$ is regarded as the longitudinal spatial coordinate. The LF components of any tensor, for example, the gauge field, are similarly defined, and the metric tensor $g_{\mu\nu}$ may be read from $A^\mu B_\mu = A^+ B^- + A^- B^+ + A^\perp B^\perp$. Also $k^+$ indicates the longitudinal momentum, while $k^-$ is the corresponding LF energy. We remind that the LF Minkowski space coordinates are not related to the conventional ones, $(x^0, x^1, x^2, x^3)$, by a finite Lorentz transformation.

We follow the arguments given in ref. [8] and introduce auxiliary Lagrange multiplier fields $B(x)$, carrying the canonical dimension three. The linear gauge-fixing term $(B A_-)$ along with the ghost term $\bar{c}(-\partial \cdot D_-)c$ are added to the Lagrangian (3) such as to ensure the Becchi-Rouet-Stora [24] symmetry of the action. The relevant free field propagators are hence determined from the following bilinear terms in the action

$$\int d^2x^+ dx^- \left\{ \frac{1}{2} \left[ (F_+^-)^2 - (F_{12})^2 + 2F_{+\perp}F_{-\perp} \right] + BA_- 
+ \frac{1}{2} M^2 (2A_+ A_- - A_\perp A_\perp) + M(A_+ \partial_- \eta + A_- \partial_+ \eta - A_\perp \partial_\perp \eta)
+ (\partial_+ \eta)(\partial_- \eta) - \frac{1}{2} \partial_+ \eta \partial_- \eta
+ (\partial_+ h)(\partial_- h) - \frac{1}{2} \partial_+ h \partial_- h - \frac{1}{2} m^2 \eta h^2 + \cdots \right\}$$

where we note that the fields $A_\perp$ as well as $B$ have no kinetic terms, and they enter in the action as auxiliary Lagrange multiplier fields.

The canonical momenta following from (4) are $\pi^+ = 0, \pi_B = 0, \pi^\perp = F_{-\perp}, \pi_- = F_{+\perp} = (\partial_+ A_- - \partial_- A_+), \pi_\eta = (\partial_+ \eta + M A_-)$, and $\pi_h = \partial_- h$, which indicate that we are dealing with a constrained dynamical system. The Dirac procedure [1] will be followed in order to construct self-consistent Hamiltonian theory framework, which is useful for the canonical quantization and in the study of the relativistic invariance. The canonical Hamiltonian density is

$$H_c = \frac{1}{2} (\pi_-)^2 + \frac{1}{2} (F_{12})^2 - A_+(\partial_+ \pi_- + \partial_\perp \pi_\perp + M^2 A_+ + M \partial_- \eta) + \frac{1}{2} M^2 A_\perp A_\perp + MA_\perp \partial_\perp \eta + \frac{1}{2} \partial_+ h \partial_- h + \frac{1}{2} m_\eta^2 h^2 + \frac{1}{2} \partial_+ \eta \partial_- \eta - BA_- + \cdots$$
The primary constraints are $\pi^+ \approx 0$, $\pi_B \approx 0$ and $\chi_\perp \equiv \pi_\perp - \partial_\perp A_\perp + \partial_\perp A_\perp \approx 0$, $\chi_\eta \equiv \pi_\eta - \partial_\eta - M\partial_\eta \approx 0$, $\chi_h \equiv \pi_h - \partial_h \approx 0$ where $\approx$ stands for the weak equality relation. We now require the persistency in $\tau$ of these constraints employing the preliminary Hamiltonian, which is obtained by adding to the canonical Hamiltonian the primary constraints multiplied by undetermined Lagrange multiplier fields. In order to obtain the Hamilton’s equations of motion, we assume initially the standard Poisson brackets for all the dynamical variables present in the theory.

We are then led to the following secondary constraints

$$\Phi \equiv \partial_- \pi^- + \partial_\perp \pi^\perp + M \partial_- \eta \approx 0, \quad A_- \approx 0 \quad (7)$$

which are already present in (5) multiplied by Lagrange multiplier fields. Requiring also the persistency of $\Phi$ and $A_-$ leads to another secondary constraint

$$\Psi \equiv \pi^- + \partial_+ A_+ \approx 0. \quad (8)$$

The procedure stops at this stage, and no more constraints are seen to arise since further repetition leads to equations which would merely determine the multiplier fields.

We analyze now the nature of the LF phase space constraints derived above. In spite of the introduction of the gauge-fixing term there still survives a first class constraint $\pi_B \approx 0$, while the other ones are second class. An inspection of the equations of motion shows that we may add [6] to the set found above an additional external constraint $B \approx 0$. This would make the whole set of constraints in the theory second class. Dirac brackets satisfy the property such that we can set the constraints as strong equality relations inside them. The equal-$\tau$ Dirac bracket $\{f(x), g(y)\}_D$ which carries this property is straightforward to construct [6, 7]. Hamilton’s equations now employ the Dirac brackets rather than the Poisson ones. The phase space constraints on the light front: $\pi^+ = 0$, $A_- = 0$, $\chi_\perp = 0$, $\chi_\eta = 0$, $\chi_h = 0$, $\Phi = 0$, $\Psi = 0$, $\pi_B = 0$, and $B = 0$ thus effectively eliminate $B$ and all the canonical momenta from the theory. The surviving dynamical variables in LC gauge are found to be $h$, $\eta$ and $A_\perp$ while $A_+$ is a dependent variable which satisfies $\partial_- (\partial_+ A_+ - \partial_\perp A_\perp - M\eta) = 0$.

The canonical quantization of the theory at equal-$\tau$ is performed via the correspondence $i\{f(x), g(y)\}_D \rightarrow [f(x), g(y)]$ where the latter indicates the commutator (or anticommutator) among the corresponding field operators. The equal-LF-time commutators of the transverse components of the gauge field are found to be

$$[A_\perp(\tau, x^-, x^\perp), A_\perp(\tau, y^-, y^\perp)] = i\delta_{\perp\perp} K(x, y)$$

where $K(x, y) = -(1/4)\epsilon(x^- - y^-) \delta^2(x^\perp - y^\perp)$. The commutators are nonlocal in the longitudinal coordinate but there is no violation [25] of the microcausality principle.
on the LF. At equal LF-time, \((x - y)^2 = -(x^\perp - y^\perp)^2 < 0\), is nonvanishing for \(x^\perp \neq y^\perp\) but \(\delta^2(x^\perp - y^\perp)\) vanishes for such spacelike separation. The commutators of the transverse components of the gauge fields are physical, having the same form as the commutators of scalar fields in the front form theory. We find also

\[
\left[ \eta(\tau, x^-, x^\perp), \eta(\tau, y^-, y^\perp) \right] = iK(x, y)
\]

and some other nonvanishing ones

\[
\left[ \partial_- A_+(\tau, x^-, x^\perp), \eta(\tau, y^-, y^\perp) \right] = i M K(x, y)
\]

\[
\left[ \partial_- A_+ (\tau, x^-, x^\perp), \partial_\perp A_+ (\tau, y^-, y^\perp) \right] = i \partial_\perp K(x, y)
\]

\[
\left[ h(\tau, x^-, x^\perp), h(\tau, y^-, y^\perp) \right] = i M K(x, y)
\]

(9)

The structure of the commutators found in the LC gauge quantized theory on the LF indicates that in our framework the 't Hooft (gauge) condition, \(\partial \cdot A - M \eta = 0\), is simultaneously incorporated as an operator equation, along with the LC gauge condition \(A_- = 0\). This is parallel to the result shown [8] in the earlier work on (massless) QCD where the Lorentz condition was found incorporated. It gave rise there to the doubly transverse gauge field propagator which simplified greatly the computations of loop corrections and allowed for a transparent discussion of the renormalization theory and unitarity relations in the physical LC gauge.

The reduced free LF Hamiltonian density in LC gauge, on making use of the constraints above, is shown to be

\[
H_0^{LF} = \frac{1}{2}(\partial_\perp A_+)(\partial_\perp A_-) + \frac{1}{2} M^2 A_\perp A_\perp + \frac{1}{2} (\partial_\perp \eta)(\partial_\perp \eta) + \frac{1}{2} M^2 \eta^2 + \frac{1}{2} (\partial_\perp h)(\partial_\perp h) + \frac{1}{2} m^2 h^2
\]

(11)

where the the bilinear cross term are eliminated due to the presence of the 't Hooft condition in the framework.

The Hamilton’s equations are found to lead to \((\partial^2 + M^2)A_\mu = 0, (\partial^2 + M^2)\eta = 0\) and \((\partial^2 + m^2 h^2)h = 0\). Taking into consideration the commutators among the field operators above derived we may write the momentum space expansions of the free (or interaction representation) field operators. Following the procedure parallel to that employed in ref. [8] we may write

\[
A_\mu(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2 k^\perp dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}} \sum_{(\alpha)} E_{(\alpha)}(\mu) \left[ a_{(\alpha)}(k^+, k^\perp) e^{-ik\cdot x} + a_{(\alpha)}^+(k^+, k^\perp) e^{ik\cdot x} \right]
\]

and

\[
\eta(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2 k^\perp dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}} \left[ b(k^+, k^\perp) e^{-ik\cdot x} + b^+(k^+, k^\perp) e^{ik\cdot x} \right]
\]

(12)
Here \( k^2 = M^2 \), \((\perp) = (1), (2), (\alpha) = (\perp), (3)\), \(a_{(\alpha)}(k) = a^{(\alpha)}(k)\), \(a_{(3)}(k) = -ib(k)\), and the nonvanishing commutator \([a_{(\alpha)}(k), a^{(\beta)}(l)] = \delta_{\alpha\beta} \delta^2(k_\perp - l_\perp) \delta(k^+ - l^+)\).

The three physical polarization vectors \(E_{(\perp)}^{\mu}(k) = E^{(\alpha)\mu}(k)\) of the massive gauge field (the mass arising through Higgs mechanism), satisfying \(E_{(\perp)}^{\mu}(k) = 0\), are constructed as follows. The two ones which are transverse to \(k\) may be taken to be the same as defined in the earlier work on QCD, viz,

\[
E_{(\perp)}^{\mu}(k) = E^{(\perp)\mu}(k) = -D_{\perp}^{\mu}(k)
\]

with

\[
D_{\mu\nu}(k) = D_{\nu\mu}(k) = -g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{(n \cdot k)} - \frac{k^2}{(n \cdot k)^2} n_\mu n_\nu,
\]

where the null four-vector \(n_\mu\) indicates the gauge direction, whose components have been chosen conveniently to be \(n_\mu = \delta_\mu^+\), \(n^\mu = \delta_\mu^-\). We note that \(E_{(\perp)}^{\perp}(k) = k^{\perp}/k^+,\ E_{(\perp)}^{\perp} = g_{1\perp} = -\delta_{\perp\perp}^+.\) They are also transverse to the gauge direction \(n_\mu\). The doubly transverse property [8] was very useful in the loop computations in QCD. We have

\[
\sum_{(\perp) = 1,2} E_{(\perp)}^{(\perp)\mu}(k) E_{(\perp)}^{(\perp)\rho}(k) = D_{\mu\nu}(k), \quad g^{\mu\nu} E_{(\perp)}^{(\perp)\mu}(k) E_{(\perp)}^{(\perp)\nu}(k) = g^{\perp\perp}(\perp \perp)
\]

\[
k^{\mu} E_{(\perp)}^{(\perp)\mu}(k) = 0, \quad n^{\mu} E_{(\perp)}^{(\perp)\mu}(k) = E_{(\perp)}^{(\perp)\mu}(k) = 0
\]

such that they are spacelike 4-vectors. The linearly independent third polarization vector for the massive vector boson, in our framework, is a null 4-vector being parallel to the gauge direction

\[
E^{(3)}_\mu(k) = E_{(3)\mu}(k) = \frac{M}{k^+} n_\mu, \quad q \cdot E^{(3)}(k) = -M q^+ k^+, \quad k \cdot E^{(\alpha)}(k) = -M \delta_{(\alpha)(3)}, \quad E^{(3)}(k) \cdot E^{(\alpha)}(q) = 0
\]

such that \(E_\mu^{(3)T}(k) = E_\mu^{(3)}(k) - (k \cdot E^{(3)}(k))(k_\mu/k^2) = E_\mu^{(3)}(k) + M(k_\mu/k^2)\) is transverse to \(k_\mu\). This is in contrast to the case of the conventional equal-time theory where all the three polarization vectors are spacelike [20].

The sum over the three physical polarizations is given by \(K_{\mu\nu}\)

\[
K_{\mu\nu}(k) = \sum_{(\alpha)} E^{(\alpha)\nu}(k) E^{(\alpha)\mu}(k) = D_{\mu\nu}(k) + \frac{M^2}{k^+} n_\mu n_\nu
\]

\[
= -g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{(n \cdot k)} - \frac{(k^2 - M^2)}{(n \cdot k)^2} n_\mu n_\nu,
\]

which satisfies: \(k^{\mu} K_{\mu\nu}(k) = (M^2/k^+) n_\nu\) and \(k^{\mu} k^{\nu} K_{\mu\nu}(k) = M^2\). We recall also [8]

\[
D_{\mu\lambda}(k) D^{\lambda\nu}(k) = D_{\mu\perp}(k) D^{\perp\nu}(k) = -D_{\mu\nu}(k),
\]

\[
k^{\mu} D_{\mu\nu}(k) = 0, \quad n^{\mu} D_{\mu\nu}(k) = D_{\nu\nu}(k) = 0,
\]

\[
D_{\lambda\mu}(q) D^{\mu\nu}(k) D_{\nu\rho}(q') = -D_{\lambda\mu}(q) D^{\mu\nu}(q').
\]

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The expansion of the transverse components of the gauge field is then rewritten as

\[ A_\perp(x) = -A_\perp^- = -\frac{1}{\sqrt{(2\pi)^3}} \int d^2 k^+ dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}} \left[ a_{(\perp)}(k)e^{-ik\cdot x} + a_{(\perp)}^+(k)e^{ik\cdot x} \right] \tag{21} \]

which together with the independent (would be Goldstone) field \( \eta \) describe the massive gauge field. It is convenient to define also the dependent gauge field component, \( A_+ \), by using the 't Hooft condition,

\[ \partial \cdot A|_{A_- = 0} = M\eta \]

incorporate in our LC gauge framework. We find

\[ A_+(x) = -\frac{1}{\sqrt{(2\pi)^3}} \int d^2 k^+ dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}} \left[ a_{(+)}(k)e^{-ik\cdot x} + a_{(+)\dagger}(k)e^{ik\cdot x} \right] \tag{22} \]

if \( a_{(+)} = a^{(+)} \) is defined such that

\[ k^+ a_{(+)\dagger}(k) = \left[ k_\perp a_{(\perp)}(k) - iM b(k) \right] = \left[ k_\perp a_{(\perp)}(k) + Ma_{(3)} \right]. \tag{23} \]

while we set \( a_{(-)}(k) = a^{(-)}(k) = 0 \) in view of \( A_- = 0 \). The following nonvanishing commutator is straightforward to derive

\[ \left[ a_{(\mu)}(k), a_{(\nu)\dagger}(l) \right] = K_{\mu\nu}(k)\delta^2(k_\perp - l_\perp)\delta(k^+ - l^+) \tag{24} \]

where \( \mu, \nu = -, +, \perp \). Following the standard procedure the free propagator of the massive gauge field \( A_\mu \) is found to be

\[ \langle 0|T(A_\mu(x)A_\nu(y))|0\rangle = \frac{i}{(2\pi)^4} \int d^4 k \frac{K_{\mu\nu}(k)}{(k^2 - M^2 + i\epsilon)} e^{-i\mathbf{k}\cdot(x-y)} \]. \tag{25} \]

It does not have the bad high energy behavior found in the (Proca) propagator in the unitary gauge formulation, where the would-be Nambu-Goldstone boson are absent. For \( M \to 0 \) it reduces to the doubly transverse propagator found [8] in connection with the LF quantized QCD in the LC gauge.

The Higgs field \( h(x) \) commutes with other field operators and its propagator is \( i/(k^2 - m_h^2 + i\epsilon) \). The commutation relations in (8) imply that the field \( \eta \) has an off-diagonal nonvanishing propagator with the component \( A_+ \), viz, \( \langle 0|T(\eta(x)A_+(y))|0\rangle \), while the \( \eta \eta \) propagator is given by \( i/(k^2 - M^2 + i\epsilon) \). All of the propagators in our LF framework fall off as \( 1/k^2 \). If we use the ML prescription to handle the \( 1/k^+ \) singularity along with the dimensional regularization, the general power-counting analysis also becomes [8] available. The propagators in the framework have good asymptotic behavior; the divergences encountered are no worse than in QED. The proof of perturbative renormalizability in the LC gauge in the \textit{front form} quantized framework.
presented here may be made straightforwardly along the lines performed earlier in
the conventional [23] equal-time theory. In view of the simplifying properties of \( K_{\mu\nu} \)
(and \( D_{\mu\nu} \)), the absence of ghost fields, and the availability of the power counting
rules, when we employ the dimensional regularization along with ML prescription,
the effort required in our framework is comparable, like in the case of the previous
work on QCD, to that in the conventional theory computations.

Some comments on the polarization vectors in LC gauge are in order. With the
restriction \( E_{\alpha}^{(\alpha)} = 0 \) there are only three linearly independent polarization vectors\(^4\)
as discussed above. \( E_{\mu}^{(\perp)}(k) \) are transverse with respect to both \( n_\mu \) and \( k_\mu \) while
\( E_{\mu}^{(3)}(k) \) is parallel to the gauge direction \( n_\mu \) and it is equal to the sum of a transverse
component \((T)\) as well as a longitudinal one \((L)\), when referred to with respect to the
4-vector \( k_\mu \). They may be clearly defined for \( E^{(\alpha)\mu}(k) \) by

\[
E^{(\alpha)\mu}_{\mu}(k) = \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) E^{(\alpha)\nu}(k), \quad k \cdot E^{(\alpha)\mu}(k) = 0
\]

\[
E^{(\alpha)L}_{\mu}(k) = \frac{k_\mu}{k^2} \left( k \cdot E^{(\alpha)}(k) \right) = -M \frac{k_\mu}{k^2} \delta^{(\alpha)(3)},
\]

\[
k \cdot E^{(\alpha)L}(k) = k \cdot E^{(\alpha)}(k) = -M \delta^{(\alpha)(3)}, \quad E^{(\alpha)T}(k) \cdot E^{(\beta)L}(k) = 0 \tag{26}
\]
such that \( E_{\mu}^{(\perp)T}(k) = E_{\mu}^{(\perp)}(k), \ E_{\mu}^{(3)L}(k) = -M (k_\mu/k^2) \ E_{\mu}^{(3)T}(k) = M (k_\mu/k^2 - n_\mu/k^+), \) and \( E_{\perp}^{(3)L,T}(k) \neq 0. \)

The following analogous decomposition of \( K_{\mu\nu} \) is useful in computations

\[
K_{\mu\nu}(k) = K_{\mu\nu}^{T}(k) + K_{\mu\nu}^{L}(k) \tag{27}
\]

where\(^5\)

\[
K_{\mu\nu}^{L}(k) = \left( \frac{M^2}{k^2} \right) d_{\mu\nu}(k)
\]

\[
K_{\mu\nu}^{T}(k) = K_{\mu\nu}(k) - K_{\mu\nu}^{L}(k) = D_{\mu\nu}(k) + M^2 \left( \frac{n_\mu n_\nu}{(n \cdot k)^2} - \frac{d_{\mu\nu}(k)}{k^2} \right)
\]

\[
= \left( k^2 - M^2 \right) \left[ \frac{d_{\mu\nu}(k)}{k^2} - \frac{n_\mu n_\nu}{k^+} \right] \tag{28}
\]

where

\[
d_{\mu\nu}(k) = -g_{\mu\nu} + n_\mu k_\nu + n_\nu k_\mu, \quad k^\mu d_{\mu\nu}(k) = \frac{k^2}{k^+} n_\nu, \quad k^\alpha k^\beta d_{\mu\nu}(k) = k^2 \tag{29}
\]

\(^4\)It is easily shown that \( n_\mu, n_\nu, E_{\mu}^{(\perp)}(k) \), where \( n_\mu = \delta_\mu^+ \) is the null vector dual to \( n_\mu = \delta_\mu^- \) constitute a convenient basis for 4-vectors in our context.

\(^5\)\( K_{\mu\nu}^{L}(k) \neq \sum_{(3)} E^{(\alpha)L}_{\mu}(k) E^{(\alpha)L}_{\nu}(k). \)
They are symmetric and some interesting properties are

\[ K^L_{\mu} - (k) = K^T_{\mu} - (k) = 0, \quad k^\mu K^T_{\mu\nu}(k) = 0, \quad k^\mu K^L_{\mu\nu}(k) = k^\mu K^L_{\mu\nu}(k) = (M^2 / k^+) n_\nu, \quad k^\mu k^\nu K^L_{\mu\nu}(k) = M^2. \]

From the properties of \( D^{\mu\nu}(k) \) we easily derive

\[ K_{\mu\nu}(k) K^\rho_{\nu}(k) = d_{\mu\nu}(k) d^\rho_{\nu}(k) = - D_{\mu\nu}(k) \quad (30) \]

and

\[ K^L_{\mu\rho}(k) K^T_{\nu\rho}(k) = - \frac{M^2 (k^2 - M^2)}{(k^2)^2} D_{\mu\nu}(k), \]
\[ K^L_{\mu\rho}(k) K^L_{\nu\rho}(k) = - \frac{M^4}{(k^2)^2} D_{\mu\nu}(k), \]
\[ K^T_{\mu\rho}(k) K^T_{\nu\rho}(k) = - \frac{(k^2 - M^2)^2}{(k^2)^2} D_{\mu\nu}(k) \quad (31) \]

For completeness we note that

\[ \sum_{(\alpha)} [E^{(\alpha)L}_{\mu} E^{(\alpha)L}_{\nu} + E^{(\alpha)L}_{\mu} E^{(\alpha)T}_{\nu} + E^{(\alpha)T}_{\mu} E^{(\alpha)L}_{\nu}] = K^L_{\mu\nu}(k) + \frac{M^2}{k^2} (g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \quad (32) \]

while

\[ \sum_{(\alpha)} E^{(\alpha)T}_{\mu}(k) E^{(\alpha)T}_{\nu}(k) = K^T_{\mu\nu}(k) - \frac{M^2}{k^2} (g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \quad (33) \]

### 2.1 Interaction Hamiltonian

The interaction Hamiltonian, in LC gauge \( A_\mu = 0 \), is derived to be

\[ \mathcal{H}_{\text{int}} = \mathcal{L}_{\text{int}} = e M A_\mu A^\mu h - \frac{e m_h^2}{2 M} (h^2 + h^2) h + e (h \partial_\mu \eta - \eta \partial_\mu h) A^\mu + \frac{e^2}{2} (h^2 + \eta^2) A_\mu A^\mu - \frac{\lambda}{4} (\eta^2 + h^2)^2 + e^2 \left( \frac{1}{\partial_-} j^+ \right) \left( \frac{1}{\partial_-} j^+ \right) \quad (34) \]

The last term here is the additional instantaneous interaction in LC gauge and \( j_\mu = (h \partial_\mu \eta - \eta \partial_\mu h) \).

### 3 GSW model of electroweak interactions

#### 3.1 SSB of \( SU(2) \otimes U(1) \) local gauge symmetry

A condensed review of the GWS model will be given below to define our notation. The model constructs a unified description of the electromagnetic and weak interactions
by employing the spontaneously broken gauge theory based on the nonabelian gauge group $SU_W(2) \otimes U_Y(1)$, the direct product of the weak isospin and the abelian hypercharge groups. The corresponding hermitian generators are $t_a = (\vec{t}, t_Y)$, where $a = (1, 2, 3, Y)$, $\vec{t} = (t_1, t_2, t_3)$, $t_Y = Y I$. Here $\vec{t}$ are isospin generators, $I$ is the identity matrix and $Y$ indicates the hypercharge. For the spontaneous breaking a complex scalar field, Higgs doublet $\Phi$, in the iso-spinor representation, with $t = 1/2$, $\vec{t} = \vec{\sigma}/2$, is introduced

$$\Phi = \begin{pmatrix} G^+ \\ \chi^o \end{pmatrix}$$  \(35\)

The value $Y(\Phi) = 1/2$ is assigned to it by convention such that the upper component $G^+$ corresponds to the unit eigenvalue of the $(U(1)_{em}$ or Charge) generator $Q = (t_3 + Y)$ and the lower one to the value zero. Under $SU_W(2) \otimes U_Y(1)$ it transforms as

$$\Phi(x) \rightarrow e^{ig\vec{t}\vec{\alpha}(x)} e^{ig'Y I \alpha_Y(x)} \Phi(x)$$  \(36\)

where $g$ and $g'$ indicate the two gauge coupling constants while $\alpha_a(x)$ are the gauge transformation parameters. The gauge covariant derivative may be defined as

$$D_\mu = (I \partial_\mu - ig \vec{A}_\mu \cdot \vec{t} - ig'Y I B_\mu)$$  \(37\)

where $\vec{A}_\mu$ and $B_\mu$ are real valued gauge fields.

The nonabelian gauge theory Lagrangian is written as

$$L = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + (D_\mu \Phi)^\dagger D_\mu \Phi - V(\Phi^\dagger \Phi)$$  \(38\)

where the gauge invariant scalar potential contains, at most, quartic terms in $\Phi$, so that the theory is renormalizable

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2$$  \(39\)

where $\lambda > 0$ and $\mu^2 < 0$. The gauge field strengths are $F^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu$ where $a, b, c = 1, 2, 3$ are the $SU(2)$ gauge group indices, $f_{abc} \equiv \epsilon_{abc}$, while $F^{Y}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$.

The description [17] of SSB in the abelian case (Appendix A) can be extended to the nonabelian one straightforwardly. It may be shown [18] here too that none of the symmetry generators break the LF vacuum symmetry but the expression which counts the number of goldstone bosons is found to be identical to the one in the conventional theory [20]. The description [18] of the tree level Higgs mechanism is easily extended for the nonabelian gauge group.

It is again convenient here as well to introduce real fields $h, \phi_1, \phi_2, \phi_3 \equiv G^o$ which have vanishing vacuum expectation values and write

$$G^+ \equiv -i \phi^- = -\frac{i}{\sqrt{2}} (\phi_1(x) - i \phi_2(x))$$

$$\chi^o = \frac{v}{\sqrt{2}} + \frac{1}{\sqrt{2}} (h(x) + i G^o(x))$$  \(40\)
where $v = \sqrt{-\mu^2/\lambda}$. In other words $\Phi = \Phi_{cl} + \varphi$ such that

$$
\Phi_{cl} \equiv <0|\Phi|0> = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}
$$

which is taken to be the classical vacuum configuration\(^6\) in the SSB case when $\mu^2 < 0$. This parametrization of $\Phi_{cl}$ can always be assumed if we make use of the (global) symmetry of the action under $SU_W(2)$ and $U_Y(1)$. We verify that $t_a \Phi_{cl} \neq 0$ but $Q \Phi_{cl} \equiv (t_3 + Y) = 0$ where the linear combination $Q$ is the generator of the unbroken residual $U(1)_{em}$ symmetry. We note also that $\Phi^\dagger \Phi = (\phi_1^2 + \phi_2^2 + \phi_3^2 + \sigma^2)/2$ where, $\sigma = (v + h(x))$. The potential $V$ above is invariant under the larger $O(4) \approx SU(2) \times SU(2)$ symmetry, which is broken by the field $\sigma$ when it acquires a non zero vacuum expectation value.

The gauge field combinations $(W^\pm_\mu, Z)$ and photon $A_\mu$ (see below) are useful

$$
W^\pm_\mu = \frac{1}{\sqrt{2}} (A^1_\mu \pm i A^2_\mu)
$$

$$
Z_\mu = (A^3_\mu \cos \theta_W - B_\mu \sin \theta_W)
$$

$$
A_\mu = (B_\mu \cos \theta_W + A^3_\mu \sin \theta_W)
$$

(42)

Here $\theta_W$ is the Weinberg angle such that $g \sin \theta_W = g' \cos \theta_W = e$ and $e$ is the electronic charge. The gauge covariant derivative may be conveniently re-expressed as

$$
D_\mu = \partial_\mu - i \frac{g}{\sqrt{2}} (W^+_\mu t_+ + W^-_\mu t_-) - i \frac{g}{\cos \theta_W} Z_\mu (t_3 - Q \sin^2 \theta_W) - i e Q A_\mu
$$

(43)

where $Q = (t_3 + Y)$ indicates the electric charge and $t_\pm = (t_1 \pm i t_2) = (\sigma_1 \pm i \sigma_2)/2$.

We find

$$
D_\mu \Phi = \begin{pmatrix}
\partial_\mu G^+ - i m_W W^+_\mu - i \frac{g \cos(2\theta_W)}{2 \cos \theta_W} Z_\mu + e A_\mu ] G^+ - \frac{i g}{2} W^+_\mu (h + iG^o) \\
\frac{1}{\sqrt{2}} \partial_\mu (h + iG^o) + \frac{ig}{\sqrt{2}} m_Z Z_\mu - \frac{ig}{\sqrt{2}} W^-_\mu G^+ + \frac{ig}{\sqrt{2}} \frac{1}{2 \cos \theta_W} Z_\mu (h + iG^o)
\end{pmatrix}
$$

(44)

while $(D^\mu \Phi)^\dagger D_\mu \Phi =

$$
|\partial_\mu G^+ - i m_W W^+_\mu - i \frac{g \cos(2\theta_W)}{2 \cos \theta_W} Z_\mu + e A_\mu | G^+ - \frac{i g}{2} W^+_\mu (h + iG^o)|^2 \\
+ \frac{1}{2} |\partial_\mu (h + iG^o) + ig m_Z Z_\mu - ig W^-_\mu G^+ + ig \frac{1}{2 \cos \theta_W} Z_\mu (h + iG^o)|^2
$$

(45)

\(^6\)The stability of the asymmetric solution while the instability of the symmetric one may be inferred from the study of the dynamical (partial differential ) equations of motion as usual.
Also

\[
V = \frac{1}{2} m_h^2 h^2 + 2 \lambda v \left[ G^+ G^- + \frac{1}{2} (G^{\sigma 2} + h^2) \right] h + \lambda \left[ G^+ G^- + \frac{1}{2} (G^{\sigma 2} + h^2) \right]^2
\]

\[
- \lambda \left[ G^+ G^- + \frac{1}{2} (G^{\sigma 2} + h^2) + v h + \frac{v^2}{2} + \frac{\mu^2}{2\lambda} \right]^2
\]

(46)

where we set \( m_W = g v / 2 \), \( m_Z = m_W / \cos \theta_W \) indicating the vector boson masses. Interaction vertices are the cubic and quartic terms in these expressions. For example, the cubic higgs boson interaction with charged vector bosons is

\[
\left[ g m_W W^\pm W^{\pm \mu} - \frac{g}{2} \left[ (\partial_\mu G^-) W^{\pm \mu} - (\partial^\mu G^+) W^- \right] + 2 \lambda v G^+ G^- \right] h.
\]

(47)

The quadratic terms in the bosonic Lagrangian which define the free theory are

\[
\frac{1}{2} (\partial^\mu h) \partial_\mu h - \frac{1}{2} m_h^2 h^2 + \frac{1}{2} m_Z^2 Z_\mu Z^\mu + \frac{1}{2} (\partial^\mu G^0) \partial_\mu G^0 + m_Z Z_\mu \partial^\mu G^0
\]

\[
+ (\partial_\mu G^+) \partial^\mu G^+ + m_W^2 W^\pm \partial^\mu G^0 + m_W \left[ (\partial_\mu G^+) W^\pm - (\partial^\mu G^+) W^\pm + (\partial^\mu G^+) \right]
\]

(48)

No mass terms arise for the (goldstone) fields \( G^\pm \) and \( G^0 \) or for the photon field \( A_\mu \). We note that \( (m_h/m_W)^2 = 8 \lambda / g^2 \) and \( m_h^2 / m_W = (4 \lambda v / g) \), \( m_W^2 / (m_Z \cos^2 \theta_W) = 1 \), \( \sqrt{2} = (\sqrt{8} G_F)^{(1/2)} \approx 174 \text{ GeV} \), and \( G_F / \sqrt{2} = g^2 / (8 m_W^2) = 1/(2 v^2) \). The bilinear terms may be rewritten as

\[
-\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2
\]

\[
-\frac{1}{4} (\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2 + \frac{1}{2} m_Z^2 Z_\mu Z^\mu + \frac{1}{2} (\partial^\mu G^0) \partial_\mu G^0 + m_Z Z_\mu \partial^\mu G^0
\]

\[
-\frac{1}{4} (\partial_\mu A^\mu_\nu - \partial_\nu A^\mu_\mu)^2 + \frac{1}{2} m_W^2 A^\mu_\nu A^{\mu \nu} + \frac{1}{2} (\partial^\mu \phi_1) \partial_\mu \phi_1 + m_W A^\mu_\nu \partial^\mu \phi_1
\]

\[
-\frac{1}{4} (\partial_\mu A^\mu_\nu - \partial_\nu A^\mu_\mu)^2 + \frac{1}{2} m_W A^\mu_\nu A^{\mu \nu} + \frac{1}{2} (\partial^\mu \phi_2) \partial_\mu \phi_2 + m_W A^\mu_\nu \partial^\mu \phi_2
\]

\[
+\frac{1}{2} (\partial^\mu h) \partial_\mu h - \frac{1}{2} m_h^2 h^2
\]

(50)

In order to quantize the free theory in LC gauge, \( A_- = Z_- = W^\pm = 0 \) we may simply take over the discussion given in Sec. 2 on abelian Higgs theory and the one

\[
\frac{1}{\sqrt{2}} (F^\mu_\nu + i F^\mu_\nu) = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu \pm i g \left( W^\pm_\mu A^3_\nu - W^\pm_\nu A^3_\mu \right)
\]

\[
F^\mu_\nu = \left[ (\partial_\mu A_\nu - \partial_\nu A_\mu) \cos \theta_W - (\partial_\mu Z_\nu - \partial_\nu Z_\mu) \sin \theta_W \right]
\]

\[
(\partial_\mu A^3_\nu - \partial_\nu A^3_\mu) = \left[ (\partial_\mu Z_\nu - \partial_\nu Z_\mu) \cos \theta_W + (\partial_\mu A_\nu - \partial_\nu A_\mu) \sin \theta_W \right]
\]

(49)
given in the earlier paper [8] on QCD for the massless gauge field. For comparison, we recall that quantization in the $R_\xi$ gauges in the conventional framework requires us to add also the ghosts fields which bring in also their interactions with the Higgs and other physical fields. Also, the parameter $\xi$ may be different for the $W_\mu$, $Z_\mu$, and $A_\mu$. It is required also to show that the physical amplitudes are independent of these parameters. Their are no such couplings in our framework, since there are no ghosts or they decouple in LC gauge.

The 't Hooft conditions in the massive case read as: $\partial \cdot W^\pm = \pm im_W G^\pm$, $\partial \cdot Z = m_Z G^0$ while for the massless field we obtain [8] the Lorentz condition $\partial \cdot A = 0$. The momentum space expansions of the quantized field operators are easily found to be

\[
A^\mu(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2k^+ dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}} \sum_{\alpha=(1),(2)} E_\alpha^\mu(k) \left[ a_\alpha(k^+, k^+) e^{-ik \cdot x} + a_\alpha^+(k^+, k^+) e^{ik \cdot x} \right]
\]

\[
W^\mu_\nu(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2k^+ dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}} \sum_{\alpha=\mu} E_\alpha^\mu(k) \left[ a_\alpha^W(k^+, k^+) e^{-ik \cdot x} + a_\alpha^W(k^+, k^+) e^{ik \cdot x} \right]
\]

\[
Z^\mu(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2k^+ dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}} \sum_{\alpha=\nu} E_\alpha^\mu(k) \left[ a_\alpha^Z(k^+, k^+) e^{-ik \cdot x} + a_\alpha^Z(k^+, k^+) e^{ik \cdot x} \right]
\]

(51)

For completeness we collect here the cubic and quartic self interactions of the gauge fields arising from the $F^a_{\mu\nu} F^{a\mu\nu}$ term

\[
ig \left[ (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) W^-\mu - (\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) W^+\mu \right] A^{3\nu}
\]

\[
+ ig W^\mu_\nu \left( \partial_\mu A^{3\nu} - \partial_\nu A^{3\mu} \right)
\]

\[
+ g^2 \left[ \frac{1}{4} (W^\mu_\nu W_\nu^+ - W_\mu^+ W^\nu) A_\mu \right] A^{3\rho} A^{3\sigma} (g^{\rho\sigma} g^{3\nu} - g^{\rho\nu} g^{3\sigma})
\]

(52)

where $A^3_\mu = [A_\mu \sin \theta_W + Z_\mu \cos \theta_W]$. We remind, however, that the complete $W^+W^-\gamma$ coupling, for example, requires in addition the interaction terms carrying $G^\pm$ fields, arising from $|D_\mu \Phi|^2$ term as well.

### 3.2 Fermionic sector

The fermionic matter content of GWS model has three generations with each one containing quarks and leptons. The left-handed components of the fermion fields are assigned to the iso-spinor representation while the right-handed to the singlet of $SU(2)_W$. For example, in the first generation with quarks ($u, d$) and leptons ($\nu_e, e^-$) we make the following assignments

\[
\psi_L: \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \in \ t = \frac{1}{2}; \quad \begin{pmatrix} u_R, d_R, e_R \end{pmatrix} \quad \in \ t = 0
\]

(53)
Here \( \psi_L = [(1 - \gamma_5)/2] \psi, \bar{\psi}_L = \bar{\psi}_L [(1 + \gamma_5)/2] \), \( \psi_R = [(1 + \gamma_5)/2] \psi, \gamma_5 = \gamma_5^1, \gamma_5^2 = I \) etc. Each left-handed doublet is assigned a value of the hypercharge \( Y \) like for the Higgs doublet. For example, \( Y(u_R) = Q(u_R) = Q(u_L) = (Y + 1/2) \) and \( Q(d) = (Y - 1/2) = Y(d_R) \), where \( Y = Y(u_L) = Y(d_L) \). We recall \( Y(e^-) = -1/2 \) and \( Y(e^+) = 1/2 \).

We base our discussion below on a single pair of generic fields \( \psi \equiv (u, d)^T \) with its left-handed components carrying the hypercharge \( Y \). It may stand for \((\nu_e, e^-)\), \((t, b)\), \((c, s)\), etc. The gauge invariant weak interaction Lagrangian for massless fermions may be written as

\[
\bar{\psi}_L i \gamma^\mu D^\mu \psi_L + \bar{u}_R i \gamma^\mu D^\mu u_R + \bar{d}_R i \gamma^\mu D^\mu d_R
\]  

(54)

The assignments of the chiral components to distinct representations of \( SU_W(2) \) and the requirement of the gauge invariance do not allow to introduce directly the fermionic mass terms in the Lagrangian. Such terms may, however, be generated through SSB if the following gauge invariant Yukawa interaction is added to the theory

\[-\lambda_d (\bar{\psi}_L \Phi) d_R - \lambda_u (\bar{\psi}_L i \sigma_2 \Phi^*) u_R + h.c. \]  

(55)

Here \( \lambda_u, \lambda_d \), are real couplings, without any connection with the weak interaction coupling constant, and we used \( Y(\Phi^*) = -1/2 \). We find the generation of the mass terms: \(- (m_u \bar{u} u + m_d \bar{d} d) \), where we set \( \lambda_d v = \sqrt{2} m_d \), \( \lambda_u v = \sqrt{2} m_u \). The Yukawa interaction terms are

\[
-\frac{g}{\sqrt{2}} \left( \frac{m_d}{m_W} \right) \left[ \bar{u} \frac{(1 + \gamma_5)}{2} d G^+ + \bar{d} \frac{(1 - \gamma_5)}{2} u G^- + \frac{1}{\sqrt{2}} \bar{d} d h + \frac{i}{\sqrt{2}} \bar{d} \gamma_5 d G^0 \right] \\
-\frac{g}{\sqrt{2}} \left( \frac{m_u}{m_W} \right) \left[ -\bar{\psi} \frac{(1 - \gamma_5)}{2} d G^+ - \bar{d} \frac{(1 + \gamma_5)}{2} u G^- + \frac{1}{\sqrt{2}} \bar{\psi} u h - \frac{i}{\sqrt{2}} \bar{\psi} \gamma_5 u G^0 \right]
\]

(56)

adding thereby additional parameters in the model.

Besides the Yukawa interaction term the full fermionic Lagrangian contains the following terms

\[
\bar{\psi} \left[ i \gamma^\mu (\partial_\mu - ieQ(u)A_\mu) - m_u \right] u + \bar{d} \left[ i \gamma^\mu (\partial_\mu - ieQ(d)A_\mu) - m_d \right] d \\
+ g \left( W^+_\mu J^{\mu+}_W + W^-_\mu J^{\mu-}_W + Z_\mu J^{\mu}_Z \right)
\]

(57)

where

\[
J^{\mu+}_W = \frac{1}{\sqrt{2}} (\bar{\psi}_L \gamma^\mu t_+ \psi_L) = \frac{1}{2 \sqrt{2}} \bar{u} \gamma^\mu (1 - \gamma_5) d \\
J^{\mu-}_W = \frac{1}{\sqrt{2}} (\bar{\psi}_L \gamma^\mu t_- \psi_L) = \frac{1}{2 \sqrt{2}} \bar{d} \gamma^\mu (1 - \gamma_5) u \\
J^{\mu}_e = Q(u) \bar{\psi} \gamma^\mu u + Q(d) \bar{\psi} \gamma^\mu d
\]
\[ J_Z^\mu = \frac{1}{\cos \theta_W} \left[ \bar{\psi}_L \gamma^\mu t_3 \psi_L - \sin^2 \theta_W J_{em}^\mu \right] \]
\[ = \frac{1}{\cos \theta_W} \left[ \frac{1}{4} \bar{u} \gamma^\mu (1 - \gamma_5) u - \frac{1}{4} \bar{d} \gamma^\mu (1 - \gamma_5) d - \sin^2 \theta_W J_{em}^\mu \right] \] (58)

such that at the tree level there are no flavor changing neutral currents. The surviving \( U(1)_{em} \) gauge symmetry is also manifest.

The construction above gives the tree level description of the GWS model in terms of the set of tree level parameters \((e, m_W, m_Z, m_h, m_u, m_d)\) or alternatively \((e, \sin \theta_W, v, m_h, m_u, m_d)\). The KM matrix can be incorporated easily in our discussion.

The LF quantization is performed following the same steps as in ref. [28] (see also Appendix B). The elimination of the dependent fermionic component \( \bar{\psi}_- \) leads to instantaneous terms. However, the interaction Hamiltonian may be re-written in terms of the full spinor field \( \psi \).

### 3.3 Instantaneous interactions in GWS model

In the front form quantized GWS model in the LC gauge we generate few additional instantaneous interaction vertices among four fields. The procedure for obtaining such terms is the same as that described in ref. [8]. They arise in view of the elimination of the dependent fields \( \psi_- \) and \( A_+ \), in the context of Dyson-Wick expansion, which uses interaction representation, and the requirement to separate the full Hamiltonian into a free and an interaction part. We illustrate it in Appendix B by considering Yukawa theory where there are no gauge fields.

### 4 Illustrations

#### 4.1 Decay \( h \to W + W \)

This decay is interesting also in connection with the goldstone boson or electroweak equivalence theorem. It is clear from the expressions of the relevant interaction vertices in Sec. 2 and Sec. 3 that it suffices to consider the abelian theory. The \( AA h \) interaction term gives the decay into two transverse vector bosons. The matrix element is

\[ \mathcal{M}_1 = (ieM) 2 E^{(\alpha)}(k) \cdot E^{(\beta)}(k') = -2ieM E_{\perp}^{(\alpha)}(k) E_{\perp}^{(\beta)}(k') \] (59)

where \( P_\mu = k_\mu + k'_\mu \) is the 4-momentum of the Higgs particle. The \( \eta^2 h \) interaction term produces longitudinal bosons in the Higgs decay. The corresponding matrix

\[ \text{We follow the notation in refs. [8, 28]} \]
element is
\[ \mathcal{M}_2 = -i \frac{\lambda^\nu}{M^2} 2 (ik \cdot E^{(\alpha)}(k)) (ik' \cdot E^{(\beta)}(k')) \]
\[ = i e \frac{m_h^2}{M} \delta_{(\alpha)(3)} \delta_{(\beta)(3)} \] (60)

Finally, the \( \eta A h \) vertex gives
\[ \mathcal{M}_3 = -i e \frac{m_h^2}{M} [k^\mu k^{\nu} + k'^\mu k'^{\nu} + k^\mu k'^{\nu}] E^{(\alpha)}_{\mu}(k) E^{(\beta)}_{\nu}(k') \] (61)

The total matrix element is
\[ \mathcal{M}_{(\alpha)(\beta)} = 2 i e M \left[ g_{\mu\nu} + \frac{1}{2} \frac{m_h^2}{M^4} k_\mu k'_\nu - \frac{1}{M^2} (k^\mu k'^{\nu} + k'^\mu k'^{\nu}) \right] E^{(\alpha)}_{\mu}(k) E^{(\beta)}_{\nu}(k') \] (62)

Using mass-shell conditions we may rewrite
\[ \mathcal{M}_{(\alpha)(\beta)} = 2 i e M \left[ g_{\mu\nu} + a k^\mu k'_\nu + b (k^\mu k'^{\nu} + k'^\mu k'^{\nu}) \right] E^{(\alpha)}_{\mu}(k) E^{(\beta)}_{\nu}(k') \] (63)

where \( a = (k \cdot k')/M^4 \) and \( b = -1/M^2 \). It is straightforward to compute the sum over polarizations of the squared matrix element. We find
\[ \sum_{(\alpha)} \sum_{(\beta)} |\mathcal{M}_{(\alpha)(\beta)}|^2 = (2 e M)^2 \left[ 2 + \frac{(k \cdot k')^2}{M^4} \right] \] (64)

It agrees with the result in unitary (or Proca) gauge.

The discussion in the nonabelian theory of the Higgs decays into gauge boson pair \( W^+ W^- \) is parallel to that of the abelian theory as can be seen from the expressions \( (xxx) \), \( (xxx) \) of the corresponding higgs couplings. We need only to replace \( e \rightarrow g/2 \) and \( M \rightarrow m_W \) in the discussion above. We find
\[ \sum_{(\alpha)} \sum_{(\beta)} |\mathcal{M}_{(\alpha)(\beta)}|^2 = \frac{g^2}{4 m_W^2} \left[ m_h^4 - 4 m_h^2 m_W^2 + 12 m_W^4 \right] \] (65)

In the limit \( m_h >> m_W \) the leading term is the first one. It derives solely from \( \mathcal{M}_2 \), e.g., from the decay to the would-be goldstone particle \( \eta \), as if we set the gauge field as vanishing in the interaction Lagrangian. Similar discussions of other two body decays of the Higgs boson may be given.

The additional contributions to the matrix element coming from the would-be goldstone bosons are found manifestly displayed. The matrix element \( \mathcal{M}_2 \), which

\[ ^9 \text{We use the simplifying properties of } K_{\mu \nu}, \text{ the relation } k_\mu k'_\nu K^{\mu \nu}(k') = -M^2 + 2 (k \cdot k') k^+/k'^+, \text{ and mass-shell relations like } (k^+/k'^+ + k'^+/k'^+) \rightarrow 2(k \cdot k')/M^2, \text{ when in the c.m. frame while taking } k_\perp = k'_\perp = 0. \]
derives solely from the would-be goldstone field, receives, compared to the others, an \((m_h/m_W)^2\) enhancement factor. The result is rather general and has been given the name of the goldstone boson or electroweak equivalence theorem [21]. In the LF quantized theory it is revealed transparently and the physics of the longitudinal gauge bosons and Higgs field can be described, under certain conditions, very well in terms of the scalar self-interactions present in the initial Lagrangian while ignoring the gauge fields.

4.2 Muon Decay

The cancellation of the noncovariant terms in the previous illustration is seen easily also in the muon decay, where the noncovariant gauge propagator is involved. However, here we do require also a contribution from the instantaneous interaction.

The terms in the interaction Lagrangian density responsible for the process are read from \((xx), (xx), (xx)\)

\[
\frac{g}{2\sqrt{2}} \left[ \bar{\nu}_\mu^-(1 + \gamma_5) \left( \gamma \cdot W^+ + \frac{i m_\mu}{m_W^2} \partial \cdot W^+ \right) \right] \mu^- + \\
\bar{\mu}^- \left( \gamma \cdot W^- - \frac{i m_\mu}{m_W^2} \partial \cdot W^- \right) (1 - \gamma_5) \nu_\mu^- + (\mu \to e) + \cdots
\]

+instantaneous terms \((66)\)

where we have made use of \(G^\pm = \mp i(\partial \cdot W^\pm)/m_W\) for convenience. The matrix element for the muon decay in momentum space, excluding the instantaneous interaction contribution, reads as

\[
\left( \frac{ig}{2\sqrt{2}} \right)^2 \bar{u}(\nu_\mu) (1 + \gamma_5) \left( \gamma^\mu - \frac{m_\mu}{m_W^2} k^\mu \right) u(\mu) \frac{K_{\mu\nu}(k)}{(k^2 - m_W^2 + i\epsilon)} \times \\
\bar{u}(e) \left( \gamma^\nu - \frac{m_e}{m_W^2} k^\nu \right) (1 - \gamma_5) v(\bar{\nu}_e)
\]

On using the simplifying properties of \(K_{\mu\nu}(k)\) it reduces to (suppressing the constant factor)

\[
\bar{u}(\nu_\mu) (1 + \gamma_5) \gamma^\mu u(\mu) \frac{K_{\mu\nu}(k)}{(k^2 - m_W^2 + i\epsilon)} \bar{u}(e) \gamma^\nu(1 - \gamma_5) v(\bar{\nu}_e) \\
- \frac{m_\mu}{(k^2 - m_W^2 + i\epsilon) k^+} \bar{u}(\nu_\mu) (1 + \gamma_5) u(\mu) \bar{u}(e) \gamma^+(1 - \gamma_5) v(\bar{\nu}_e) \\
- \frac{m_e}{(k^2 - m_W^2 + i\epsilon) k^+} \bar{u}(\nu_\mu) (1 + \gamma_5) \gamma^+ u(\mu) \bar{u}(e)(1 - \gamma_5) v(\bar{\nu}_e) \\
+ \frac{m_\mu m_e}{(k^2 - m_W^2 + i\epsilon) m_W^2} \bar{u}(\nu_\mu) (1 + \gamma_5) u(\mu) \bar{u}(e) \gamma^+(1 - \gamma_5) v(\bar{\nu}_e)
\]

\((68)\)
Consider the contributions from the first term. The noncovariant ones carrying $1/k^+$
dependence cancel the second and the third terms. There is also an instantaneous
contribution
\[ -\frac{1}{k^{+2}} \bar{u}(\nu_o) (1 + \gamma_5) \gamma^+ u(\mu) \bar{u}(e) \gamma^+ (1 - \gamma_5) v(\nu_e) \]  
(69)

It gets compensated by the additional instantaneous interaction terms in our LC
gauge framework. The final result agrees with the covariant one in the unitary gauge.

### 4.3 Decay $t \rightarrow b + W^+$

The relevant interaction terms in the present case are
\[ \frac{g}{2\sqrt{2}} \bar{b} \left[ \gamma \cdot W^-(1 - \gamma_5) + \left( \frac{m_t - m_b}{m_W} + \frac{m_t + m_b}{m_W} \right) \gamma^+ \right] _{\mu} G^\pm = t + h.c. \]  
(70)

The matrix element may be written as
\[ \frac{ig}{2\sqrt{2}} \bar{u}^{(r)}(b) \left[ \gamma^\mu (1 - \gamma_5) - \frac{m_t}{m^2_W} k^\mu (1 + \gamma_5) \right] u^{(r)}(t) E^{(a)}_{\mu} \]
\[ = \frac{ig}{2\sqrt{2}} \bar{u}^{(r)}(b) \left[ \gamma^\mu E^{(a)}_{\mu}(k) (1 - \gamma_5) + \frac{m_t}{m^2_W} \delta_{(a)(3)} (1 + \gamma_5) \right] u^{(r)}(t) \]  
(71)

Here we have set $m_b = 0$ for simplicity and we recall that $(a) = (\perp), (3)$ indicate
the three polarization states of the massive vector boson as discussed in Sec. 2. For
spinor field we follow the notation of ref. [8]. The $m_t$ enhancement of the matrix
element containing solely the would-be goldstone bosons $G^+$ is similar to that in the
the Higgs decay described above. It is another illustration of the electroweak equiv-

cence theorem. Since the Higgs boson couples to fermion mass the heavy fermion
contributions do not decouple. The sum over spins and polarizations of the squared
invariant matrix element here is found to be proportional to
\[ \left[ q^\mu p^\nu + q^\nu p^\mu - (q \cdot p) g^{\mu\nu} + \left( \frac{m_t}{m^2_W} \right)^2 \left( q \cdot p \frac{k^\mu k^\nu}{m^2_W} - q^\mu k^\nu - q^\nu k^\mu \right) \right] K_{\mu\nu}(k) \]
\[ = \left[ q^\mu p^\nu + q^\nu p^\mu - (q \cdot p) g^{\mu\nu} \right] d_{\mu\nu}(k) + \left( \frac{m_t}{m^2_W} \right)^2 \left( q \cdot p - 2m^2_W \frac{q^+}{k^+} \right) \]  
(72)

where the mass-shell conditions like $2k \cdot q = (m_t^2 - m^2_W)$, $2k \cdot p = (m_b^2 + m^2_W)$, $q^2 = 0$
have been used. Collecting together the noncovariant terms we rewrite it as
\[ = -\left[ q^\mu p^\nu + q^\nu p^\mu - (q \cdot p) g^{\mu\nu} \right] g_{\mu\nu} + \frac{1}{k^+} \left( 2q \cdot k p^+ + 2k \cdot p q^+ - 2q \cdot p - 2m_t^2 q^+ \right) \]
\[ + \left( \frac{m_t}{m_W} \right)^2 q \cdot k = -\left[ q^\mu p^\nu + q^\nu p^\mu - (q \cdot p) g^{\mu\nu} \right] g_{\mu\nu} + \left( \frac{m_t}{m_W} \right)^2 q \cdot k \]
\[ = \left(-g_{\mu\nu} + \frac{k^\mu k^\nu}{m^2_W} \right) \left[ q^\mu p^\nu + q^\nu p^\mu - (q \cdot p) g^{\mu\nu} \right] \]  
(73)
The noncovariant terms cancel out giving the covariant result of the unitary gauge$^{10}$. 

Appendix A

Spontaneous symmetry breaking description on the LF

The following discussion on SSB may be given for any gauge group $^{17, 22}$. For simplicity and its direct relevance to Sec. 1 we consider the scalar theory Lagrangian with global $U(1)$ symmetry

$$\mathcal{L} = \partial_+ \phi^\dagger \partial_- \phi + \partial_- \phi^\dagger \partial_+ \phi - \partial_\perp \phi^\dagger \partial_\perp \phi - V(\phi^\dagger \phi)$$ (74)

where $V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$ with $\lambda > 0$ and $\mu^2 < 0$. To canonically quantize the theory we must construct a Hamiltonian framework for the constrained dynamics described by the above Lagrangian. The Dirac procedure $^6$ is convenient to use. Before applying it, however, we make $^{17}$ the separation

$$\phi(x^+, x^-, x^\perp) = \omega(x^+, x^\perp) + \varphi(x^+, x^-, x^\perp).$$

The field $\varphi$ indicates the quantum fluctuations above the dynamical (condensate) variable $\omega$, associated with the zero-longitudinal momentum-mode. The LF Hamiltonian framework is found to contain in it also a (second class) constraint equations $^{17}$, which relates the condensate variables with the fluctuation fields. The variable $\omega$ is shown $^{17, 22}$ to have vanishing Dirac brackets with itself and with $\varphi$. It is thus a c-number (background field) in the quantized theory$^{12}$ The constraint equations$^{13}$ in the present case are

$$\int d^2x^+ dx^- [\partial_\perp \partial_\perp \phi - \frac{\delta V}{\delta \phi^\dagger}] = 0,$$

$$\int d^2x^+ dx^- [\partial_\perp \partial_\perp \phi^\dagger - \frac{\delta V}{\delta \phi}] = 0.$$ (75)

In the discussion to follow we consider only the case where $\partial_\perp \omega = 0$. At the classical (tree) level, since the fluctuations $\varphi$ are assumed bounded, it is shown $^{17}$ to follow that $\frac{\delta V}{\delta \phi}|_{\phi = \omega} = \frac{\delta V}{\delta \phi^\dagger}|_{\phi = \omega} = 0$. This coincides with the result in the conventional equal-time framework. It is obtained there after imposing additional constraints, which are based on physical considerations (seemingly not available or evident on the LF). The possible values of $\omega$ are $\omega = 0$ or $\omega^\dagger \omega = -\frac{\mu^2}{2\lambda}$. The stability of these

$^{10}\Gamma = \frac{G_F m_t^2}{8\sqrt{2}\pi} (1 - \frac{m_t}{m_t})^2 \left(1 + 2 \frac{m_\nu}{m_t}\right).$

$^{11}$Such a decomposition may also be shown to follow $^{17}$ as an external $^6$ gauge-fixing condition, corresponding to a first class constraint in the theory, when we apply the Dirac procedure. We note that $\int d^2x^+ dx^- \varphi = 0$ such that $\varphi$ has vanishing zero-longitudinal momentum-mode.

$^{12}$In the bosonized Schwinger model it is shown $^{27}$ to be a q-number operator and where its presence gives rise to the chiral and $\theta$-vacua.

$^{13}$They may also be obtained by integrating the Lagrange equations but we must construct LF Hamiltonian frame to canonically quantize the theory $^{17}$. 

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solutions may be studied as usual from the Lagrange equations; the nonvanishing $\omega$ gives rise to stable solutions in the Nambu-Goldstone phase under study. The (classical) vacuum state is degenerate and characterized by a fixed value of $\omega = \sqrt{-\mu^2/(2\lambda)} e^{i\delta}$ where $\delta$ is real and arbitrary. In view of the invariance of the action under the phase symmetry transformations: $\varphi \rightarrow e^{i\alpha} \varphi$, $\omega \rightarrow e^{i\alpha} \omega$, we may, without any loss of generality, conveniently assume $\omega \equiv v/\sqrt{2}$ where $v = \sqrt{-\mu^2/\lambda}$ is a fixed real constant. A phase transformation would not leave invariant this classical vacuum state and the symmetry is said to be broken spontaneously (see also Sec. 3.1).

At the quantum level, on the other hand, the LF field theoretic generator of $U(1)$ symmetry annihilates the LF vacuum state independent of the broken symmetry or not. The symmetry transformations always leave the LF vacuum invariant while the SSB is manifested, for example, in the non-conservation of some of the symmetry currents [18, 22]. These conclusions are true in general.

Dirac procedure is straightforward to apply and the quantized theory is obtained by invoking the correspondence of the Dirac brackets with the commutators of the corresponding quantized field operators. In the LF quantized theory we find the following non-vanishing equal-$x^+$ commutator

$$[\varphi(x^+, x^-, x^\perp), \varphi(y^+, y^-, y^\perp)] |_{x^+ = y^+} = -\frac{i}{4} \epsilon(x^- - y^-) \delta^2(x^\perp - y^\perp)$$

which does not violate the principle of microcausality on the LF, in spite of the nonlocality present in it along the $x^-$ direction. The hermitian symmetry generator is constructed straightforwardly

$$G(x^+) = \int d^2x^\perp dx^- j_-, \text{ where } j_\mu = i \left[ \varphi^\dagger \partial_\mu \varphi - \varphi \partial_\mu \varphi^\dagger \right]$$

such that $[\varphi(x), G] = \varphi$, $[\varphi(x)^\dagger, G] = -\varphi^\dagger$. The on-shell conserved Noether symmetry current is given by

$$J_\mu = i \left[ \phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger \right], \quad \partial_\mu J^\mu = 0$$

which shows that the symmetry current ($\phi = v/\sqrt{2} + \varphi$)

$$j_\mu = J_\mu - \frac{i v}{\sqrt{2}} (\partial_\mu \varphi - \partial_\mu \varphi^\dagger)$$

$$\partial^\mu j_\mu = \frac{i v}{\sqrt{2}} \partial^2 (\varphi - \varphi^\dagger)$$

is not conserved in the broken phase. In the LF quantized theory, the two currents $j_\mu$ and $J_\mu$, however, give rise to the same charge or generator, if the surface terms may
be ignored. The field theoretic generator $G$ generates the symmetry transformations of the field operators\(^{14}\)

The LF commutator may be realized by the following momentum space expansion

$$
\varphi(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2k^\perp dk^+ \frac{\theta(k^+)}{\sqrt{2k^+}} \left[ a(k)e^{-ik\cdot x} + b^\dagger(k)e^{ik\cdot x} \right]
$$

(80)

where the nonvanishing commutators are $[a(k), a^\dagger(l)] = [b(k), b^\dagger(l)] = \delta^2(k^\perp - l^\perp)$, $\delta(k^+ - l^+)$. The symmetry generator in momentum space is found to be

$$
G = \int d^2k^\perp dk^+ \theta(k^+) \left[ a^\dagger(k)a(k) - b^\dagger(k)b(k) \right].
$$

(81)

In the LF quantized theory only this term is present. It is already normal ordered and annihilates the LF vacuum. This is in contrast to the case of equal-time quantized conventional theory, where there is an additional term in the symmetry generator which does not annihilate the corresponding conventional vacuum state. The LF vacuum thus remains invariant under the symmetry transformations independent of the SSB in the theory. The broken symmetry manifests [18] itself in the non-conservation of (some) symmetry currents or in the operator LF Hamiltonian and the criterion for the counting of the Goldstone bosons is shown [18] to be the same as in the conventional theory description. In the conventional framework the broken symmetry generators do not annihilate the vacuum, which breaks the corresponding symmetry spontaneously. The description of the tree level Higgs mechanism on the LF is discussed in ref. [18]. Sec. 2 discusses its quantization.

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**References**


\(^{14}\)In the equal-time quantized theory where we have instead $\partial_t(\varphi - \varphi^\dagger)$ which does not drop out on the coordinate space integration. The description of the SSB hence follows [22] to be somewhat different in the two forms of the theory.


